Neutrosophic Treatment of the Static Model of Inventory Management with Deficit

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Abstract

In this article, when the inability to manage inventory is allowed in exchange for certain fines, managers can then allow the remaining stock to run out and accumulation the order until compensation is reinstated. The idea depends on the amount of stock deficit, which could be small in comparison to the storage cost. This refers to inventory models with deficits and is used to store damageable material. To manage these models, deficit costs should pair with storage costs by applying a mathematical model that illustrates the state in which deficit occurs by satisfying a requirement for one item. All that follows the principles of static modelling with a deficit and the ideal solution would be the most appropriate. We know that the ideal amount depends on the inventory demands. In the classical logic, we define the required storage amounts by the inventory cycle which has a constant value during the inventory cycle; however, that logic overlooks the fluctuation of inventory demands. Therefore, this manuscript conducts a study about static inventory models with a deficit for one material based on neutrosophic logic, which takes into consideration the indeterminate states and hence it provides more precise results, thus, it gives institutions a stable work environment with the least costs.

1. Introduction

In our daily life, Inventory is considered foundational for an enterprise and could yield high profit or significant monetary losses. Moreover, inventory volume is affected by the demand rate and recurrence of orders through time unit, whenever the demand rate is specified meticulously throughout the storage cycle according to an enterprise's requirements and at the least cost. The classic logic takes demand rate

as a constant value which does not represent reality due to the unpredictable and variable nature of demand rates. To obtain more precise results, we apply neutrosophic logic.

Neutrosophic logic is a new vision of modeling developed by American mathematician Florentine Smarandache and is designed to effectively address the uncertainties inherent in the real world, as it came to replace the binary logic, fuzzy logic, and intuitionistic logic that recognize the true and the false state of any idea, problem, and issue by introducing a third neutral state which can be interpreted as undefined or unconfirmed, indeterminate, vague, paraconsistent, and incompleteness [5,7,8,9,11]. Florentine Smarandache presented neutrosophic logic in 1995 as a generalization of fuzzy logic presented by Lotfi Zadeh [4] in 1965, also, it is an extension of intuitionistic fuzzy logic that set up by K. Atansove in 1983 [6]. In addition to that, Ahmed A. Salama presented the theory of neutrosophic classical categories as a generalization of the theory of classical categories [10, 18] and developed it, also he introduced and formulated new concepts in the fields of neutrosophic mathematics, neutrosophic statistics, neutrosophic computer science…etc. by the unfathomable tools of neutrosophic logic [15,16,17,20,27]. Neutrosophic logic has significantly grown in recent years through its application in measurement, collections, graphs, geometric programming, relational equations, topological spaces and dozens of scientific and practical fields [6,12-14,19,21-25,29-37].

This manuscript has been dedicated to the application of neutrosophic theory in one of the inventory management models, which is the static model without deficit and for a single material. As such, this application will allow us to deal with formulas of inventory management models that are expressed in a way that is uncertain and not precisely defined and take the range of values instead of a single value, as is done in the classical model. We will particularly present the static model without a deficit for a single substance according to the neutrosophic logic, when the rate of demand for stock is defined accordingly, and then we will present an applied example that showcases how to apply this logic.

2. Discussion

The responsibility of determining the ideal policy work of the institute based on its manager. Through, collecting necessary data for work and investing this data at management institute work, one of the important things that we have worked on is the inventory, because it is an important tour at producing and marketing, especially at firm's products which have warehouse has been dedicated to their equipment and sales. Inventory management is one of the most important functions of management, through determining the way of storing by allowed deficit or disallowed depend on comparing between deficit charge and storage cost, so the principal goal is determining the ideal inventory volume and its costing, which affected the firm efficiency and judge if it achieves huge earning or heavy losses, that we have studied and determined the ideal inventory volume with the minimal cost through studying static inventory models at classical logic, while when the rate of inventory requested was indeterminate, it is obvious that the rate is subject to a uniform probability distribution. thus when the rate of inventory request is an unlimited form at the time cycle we use neutrosophic uniform probability distribution, which has been presented previously [20]. All neutrosophic mathematical tools and strategies presented in this study are regarded as complementary to our previous work [28], hence anyone tracking our work will recognize that these articles are a series in which each article is complementing other articles.

2.1. The main hypotheses of the study:

1. Let Q be the demand volume.
2. The rate of the request on inventory at time unit λ_N (indeterminate), where $\lambda_{iN} = [\lambda_1, \lambda_2]$ is an interval or $\lambda_N = \{\lambda_1, \lambda_2\}$ is a set or any other indeterminate component., where λ_1 is the minimum rate of the request on inventory, λ_2 is the maximum rate of the request on inventory.
3. The constant cost for preparing request is $C_1 = K$.
4. The cost of purchase , conveyance, receive $C_2 = C.Q$.
5. The storage cost of remaining quantity at the warehouse at time unit C_3.

6. The deficit amount allowed at a storing cycle equal constant value S, the deficit cost is P for each unverified request at the time unit including delay penalty and losing customer, ...etc.

7. The time period of inventory quantity run out is $\frac{Q}{\lambda_N}$ (i.e. a storing cycle time period).

The symbol of the available storage volume at the start inventory cycle is R , symbolize to the requested volume that reaching the warehouse at the end of every cycle is Q where $R < Q$ thus the deficit value is $S = Q - R$. The available inventory volume r is connected with time by the relation: $r_t = R - \lambda_t$. The validity period of the stock $\frac{Q}{\lambda_N}$, storage cycle period $\frac{Q}{\lambda_N}$,and it is greater than $\frac{R}{\lambda_N}$. For that , we divide the first inventory cycle $[0, \frac{Q}{\lambda_N}]$ into two periods:

First period $[0, \frac{R}{\lambda_N}]$ is the work period. Second period $]\frac{R}{\lambda_N}, \frac{Q}{\lambda_N}]$ is the deficit period.

Consequently, calculating the inventory costs required calculate it through two periods, the first period represent static model calculation without any deficit, and it defined by the relation:

$$C_3 = \frac{hR^2}{2\lambda_N}$$

While the deficit costs calculate as:

$$C_4 = (-1) \int_{\frac{R}{\lambda_N}}^{\frac{Q}{\lambda_N}} P(R - \lambda_N t)dt = \frac{P(Q - R)^2}{2\lambda_N}$$

Consequently, the total cost is:

$$TC(Q, R) = K + CQ + \frac{hR^2}{2\lambda_N} + \frac{P(Q - R)^2}{2\lambda_N}$$

To have the total costs by time unit, divide $TC(Q, R)$ on the inventory cycle period $\frac{Q}{\lambda_N}$,which yields :

$$C(Q, R) = \frac{K.\lambda_N}{Q} + \lambda_N.C + \frac{hR^2}{2Q} + \frac{P(Q - R)^2}{2Q}$$

From the previous discussion, the following mathematical model can be concluded:
Find

$$\min C(Q, R) = \frac{K.\lambda_N}{Q} + \lambda_N.C + \frac{hR^2}{2Q} + \frac{P(Q - R)^2}{2Q}$$

Subject to:
$Q \geq R$, $Q \geq 0, R \geq 0$

This model is nonlinear. For finding the ideal solution, the target function is twice partially derived for Q and R, these two derived formulas can be homogeneous equations when equaling them by zeros.

$$\frac{\partial C(Q, R)}{\partial Q} = \frac{-K.\lambda_N}{Q^2} - \frac{hR^2}{2Q^2} + \frac{2P(Q - R)(2Q) - 2P(Q - R)^2}{4Q^2}$$

$$= \frac{1}{Q^2}\left[-K.\lambda_N - \frac{hR^2}{2} + P(Q - R)Q - \frac{1}{2}P(Q - R)^2\right] = 0$$

$$\frac{\partial C(Q, R)}{\partial R} = \frac{2hR}{2Q} + \frac{2P(Q - R)(-1)}{Q} = 0$$

By multiplying the above tow equations by $Q^2, 2Q$ respectively, the following two equations are gained:

$$-K.\lambda_N - \frac{hR^2}{2} + P(Q - R)Q - \frac{P(Q - R)^2}{2} = 0$$
$$2hR - 4P(Q - R) = 0$$

By solving the above system of two equations:

$$Q^* = \sqrt{\left(\frac{2K.\lambda_N}{h}\right)\left(\frac{P+h}{P}\right)} \quad , \qquad R^* = \sqrt{\left(\frac{2K.\lambda_N}{h}\right)\left(\frac{P}{P+h}\right)} \qquad (1)$$

From the equation $2hR - 4P(Q - R) = 0$, we find that R^* , Q^* having the following relationship:

$$Q^* = \frac{P+h}{P} R^*$$

To ensure that these two values verifying model conditions. We have $h > 0$, and $\frac{P+h}{P} > 0$, this implies that $Q^* \geq R^*$, then the above two solutions verify the conditions, that's mean $Q \geq R$, $Q \geq 0, R \geq 0$. So Q^*, R^* that defined in relations (1) satisfy the optimal solution of objective function. To specify the type of this objective function if it satisfy its minimum or maximum value, the Hessian matrix of the objective function is defined as below. As the variables in the nonlinear model are two, so the Hessian arrays presented as:

$$\begin{bmatrix} \dfrac{\partial^2 C}{\partial Q^2} & \dfrac{\partial^2 C}{\partial Q \partial R} \\ \dfrac{\partial^2 C}{\partial R \partial Q} & \dfrac{\partial^2 C}{\partial R^2} \end{bmatrix}$$

Calculate the derivatives and substitute its value at the point (Q^*, R^*), it is well known that hessian matrix is non negative symmetric matrix, the major mini-determinants are non-negative, hence the matrix is well defined and semi-positive, in other words, the function is concave, which implies that the optimal value is the minimum value, subsequently ,the values that we gained for (R^*, Q^*) verifies required. The total costs value is:

$$C(Q^*, R^*) = \frac{K.\lambda_N}{Q^*} + \lambda_N.C + \frac{hR^{*2}}{2Q^*} + \frac{P(Q^* - R^*)^2}{2Q^*}$$

So, the value of the total costs through the inventory cycle in which its duration $T^* = \frac{Q^*}{\lambda}$ is:

$$TC(Q^*, R^*) = K + CQ^* + \frac{hR^{*2}}{2\lambda_N} + \frac{P(Q^* - R^*)^2}{2\lambda_N}$$

and to calculate the quantity of the re-requesting, we apply the following relation:

$$Q_1 = \lambda_N d - S$$

where d is the required duration for receiving request.

2.2 Illustration [2]:

If the objective function of the constrained nonlinear programming problem is convex, and the region determined by the constraints is convex too, then, the optimal solution is the global minimum solution.

The objective function satisfies the convexity property if its Hessian matrix is well defined and positive, or, semi-defined and positive.

Any matrix will be defined and positive, if verifying the following conditions:

1.   the Matrix is symmetric.

2.   All elements of the main diameter are positive.

3.   The major mini-determinants are positive.

The matrix will be defined and semi-positive, if verifying the following conditions:

1.   the Matrix is symmetric.

2.   All elements of the main diameter are nonnegative.

3.   The major mini-determinants are nonnegative.

3. Practical Example

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This example is reformation from the classical version of the same context stated in [1] to neutrosophic version by changing the inventory rate from traditional value into neutrosophic value.

3.1 Example text by Neutrosophic logic:

Assuming we have warehouse for storing and sells material, if you know that the monthly requested rate for this material is between [0,300] ton, and the period time for receiving requested after submitting the application is five days, the cost preparing a request is (200) currency unit, the price purchasing for a ton of one material is (5000) currency unit, one ton storing cost at time unit is (10) currency unit, a deficit cost for every ton at time unit is (50) currency unit, although, the warehouse system allowed definite deficit, Find:

1- the optimal quantity for both: first request R^* , and the cyclic request Q^*.

2- the deficit amount allowed.

3- the minimum storing cost at time unit (month).

4- the minimum storing cost through storing cyclic.

Solution:

From the text of the example, we have the following data:

$\lambda_N = [0,300] , d = 5 , P = 50 , h = 10 , C = 5000 , K = 200, \lambda_N = [0,300]$

Substituting the above data in the following mathematical modal:

$$\min C(Q,R) = \frac{K.\lambda_N}{Q} + \lambda_N.C + \frac{hR^2}{2Q} + \frac{P(Q-R)^2}{2Q}$$

Subject to:

$Q \geq R , Q \geq 0, R \geq 0$

$$\min C(Q,R) = \frac{200.[0,300]}{Q} + 5000.[0,300] + \frac{10R^2}{2Q} + \frac{50(Q-R)^2}{2Q}$$

Subject to

$Q \geq R , Q \geq 0, R \geq 0$

Given that:

$$Q^* = \sqrt{\left(\frac{2K.\lambda_N}{h}\right)\left(\frac{P+h}{P}\right)}$$

$$R^* = \sqrt{\left(\frac{2K.\lambda_N}{h}\right)\left(\frac{P}{P+h}\right)} \qquad , \quad (1)$$

By substituting the given data in (1), we get

$$Q^* = \sqrt{\left(\frac{2.300.[0,300]}{10}\right)\left(\frac{50+10}{50}\right)} = \sqrt{\left(\frac{[0,180000]}{10}\right)\left(\frac{60}{50}\right)} = \sqrt{[0,21600]} \cong [0,147]$$

$$R^* = \sqrt{\left(\frac{2.300.[0,300]}{10}\right)\left(\frac{50}{50+10}\right)} = \sqrt{\left(\frac{[0,9000000]}{600}\right)} = \sqrt{[0,15000]} \cong [0,123]$$

So the deficit allowed is:

$S^* = Q^* - R^* \cong [0,147] - [0,123] = [0,24]$

Minimum storing cost at time unit (month) is:

$$C(Q^* - R^*) = \frac{200.\,[0,300]}{[0,147]} + 5000\,[0,300] + \frac{10\,([0,123)^2}{2[0,147]} + \frac{50([0,147] - [0,123])^2}{2[0,147]}$$
$$= [0,408] + [0,1500000] + [0,515] + [0,116] \cong [0,1501039]$$

To calculate total costs at storing cyclic replace with the following relation:

$$TC(Q^*, R^*) = K + CQ^* + \frac{hR^{*2}}{2\lambda_N} + \frac{P(Q^* - R^*)^2}{2\lambda_N}$$

So we have:

$$TC(Q^*, R^*) = 200 + 5000[0,147] + \frac{10\,([0,123)^2}{2[0,300]} + \frac{50([0,147] - [0,123])^2}{2[0,300]}$$
$$= 200 + [0,735000] + [0,25215] + [0,48] \cong [0,760463]$$

Calculate the net profit of one cyclic storing given that sales price for one ton is (5500) price unit, then the total sales will be:

$V = 5500.\,Q^* = 5500.\,[0,147] = [0,808500]$ (price unit by storing cyclic)

The profit through storing cyclic equals:

$\,\,B = V - TC = [0,808500] - [0,760463] = [0,48037]$ (price unit by storing cyclic)

And monthly profit equals:

$B = n^*.\,B = [0,2].\,[0,48037] = [0,96074]$ (price unit by moth)

Comparing between deficit cost and storing cost for unfulfilled request:

we find that the deficit allowed equals:

$S^* = Q^* - R^* \cong [0,147] - [0,123] = [0,24]$

for this deficit cost equal (selling price this amount _ purchase price same amount)

Vended price equal: $5500\,[0,24] = [0,132000]$;

purchase price equal: $5000\,[0,24] = [0,120000]$;

deficit cost equal: $[0,132000] - [0,120000] = [0,12000]$;

fine deficit is equal: $P\,[0,24] = 50.\,[0,24] = [0,12000]$.

It is obvious that the fine of the deficit is equal to storing cost, which means the deficit didn't occur any losses for the warehouse.

Table 1 Classical data and Neutrosophic data results

Classical data and result						Neutrosophic data and result					
$\lambda = 300$	$K = 200$	$C = 5000$	$h = 10$	$P = 50$	$d = 5$	$\lambda_N = [0,300]$	$K = 200$	$C = 5000$	$h = 10$	$P = 50$	$d = 5$
$R^* = 100\,Ton$						$R^* = [0,123]\,Ton$					
$Q^* = 120\,Ton$						$Q^* = [0,147]\,Ton$					
$S^* = 20\,Ton$						$S^* = [0,24]\,Ton$					

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$C(Q^*, R^*) = 1501000 \ unit \ price$	$C(Q^*, R^*) = [0,1501039] \ unit \ price$
$TC(Q^*, R^*) = 600400 \ unit \ price$	$TC(Q^*, R^*) = [0,760463] \ unit \ price$

4. Conclusion and results:

This manuscript presented studying for inventory models with a deficit for one material, models used for storing perishable material based on neutrosophic logic, we found that using neutrosophic models presents comprehensive studying rather than known classical studying. This studying included all definite and indefinite data side by side, we found out at indefinite value for the demand rat at inventory effects on the ideal volume for request and on total costs, and it expressible on it by approximately way where the minimum value of the optimal request volume, and presents the maximum value of optimal request volume, where enables the inventory available into deficit allowed without increasing institute losses. Subsequently, we found that existence of indeterminate affects virtually on results and this indetermination values couldn't be ignoring and excluded it from studying, because the ignoring the indeterminate values will make the nonlinear program problems lacks to more accurate results, consequently, do not getting the inventory ideal volume which meet the needs of the firm during the storing cyclic with minimal cost.

Recently, the work at classical logic isn't enough anymore where the frontiers of science foreground many new problems that need more comprehensive and accurate results than the results were having by using classic logic, and fuzzy logic. Here the neutrosophic logic comes as an essential requirement giving us a more holistic explanation for studying data and procurement accuracy requested results. Looking forwards to more studying for other management inventory models based on neutrosophic logic such as inventory s modals with safety auxiliary for one material, dynamical models and probability models and others.

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