Neutrosophic Nonlinear Models

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Abstract
Nonlinear programming is an important and essential part of operations research and is more comprehensive than linear programming, its applications have spread in all branches of science, engineering, physics, chemistry, management, economic and military fields, etc. Nonlinear programming can also be used in forecasting, estimation, applied statistics and determining the costs resulting from the production, purchase and storage of goods, the mathematical model is a nonlinear model if any of the vehicles of the target or constraints are nonlinear statements and may be nonlinear statements In both, after neutrosophic science has achieved a great development in most fields of science, it was necessary to reformulate nonlinear models according to its concepts, we present in this research a study of neutrosophic nonlinear dependencies and neutrosophic nonlinear models as a prelude to future research through which we study some methods of solving neutrosophic nonlinear models.

Keywords: Nonlinear Models; Neutrosophic Logic; Neutrosophic Nonlinear Models

1. Introduction

The mathematical model consists of decision variables and the objective function and constraints and the area of possible solutions to the nonlinear mathematical model is the set of vectors whose components achieve all constraints. The optimal solution is the vector that achieves all constraints, and the function then reaches an optimal value (maximum or minimum) [1,2,3,4] In this research, we will use the concepts of neutrosophic science. To formulate some basic concepts and definitions in nonlinear programming and to complement what was presented by researchers interested in neutrosophy, and neutrosophic science is the latest findings of logical thought in its relentless quest to absorb reality and expresses the uprising of the mind when it collides with the unreasonableness of reality or false rationality scattered by petrified systems seeking stability in a world in which the pace of development and change is accelerating and to see the stages of development of this science from its inception until today see [5]. In front of this reality, we cannot say that the possibilities available during the workflow in any facility are specific amounts, and therefore we do not guarantee that the quantities produced are specific amounts that the facility can rely on in determining the profit that it can achieve during the course of its work over time, loss or costs, so it was necessary to formulate a new mathematical method used when studying realistic problems. Consider the link of the available information about the system under study with reality and the fluctuations of the conditions surrounding the work environment, as we did for linear and dynamic programming, see [6,7,8,9]. In this paper, we present a study of nonlinear programming where we will reformulate some important definitions and concepts using the concepts of neutrosophic science.

2. Discussion

The problems of mathematical examples depend on the construction of mathematical models consisting of a goal function and constraints. These models may be linear, nonlinear or ---- models, for the goal function is either a maximization function or a minimization function of a certain quantity, this quantity depends on a number of decision variables, and they may be independent of each other or linked to each other through a set of constraints and we get values for these variables By solving the mathematical model that we get, but before that, we must identify the most important concepts and definitions related to linear functions, a thorough study of this was presented using classical values [1,2] and the results of solving nonlinear models were specific values that do not take into account the changes that may occur during the work of the systems that work according to these

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models, which may lead to large losses, which prompted us to prepare this research, through which we will reformulate some concepts of nonlinear programming using the concepts of neutrosophic science to be the basis for subsequent studies through which we present some methods of solving neutrosophic nonlinear models, which provides us with a safe working environment through the indeterminism enjoyed by neutrosophic values.

Based on the references [1,2], we present the following study:

3. Neutrosophic Mathematical Model:

In the problem of examples where the objective and constraints are in the form of neutrosophic mathematical functions, then the neutrosophic mathematical model is written in the following form:

$$Nf = Nf(x_1, x_2, --, x_n) \rightarrow (Max\ )or\ (Min)$$

According to the following restrictions:

$$Ng_i(x_1, x_2, --, x_n)\begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} Nb_i \quad ; i = 1,2, ---, m$$

$$x_1, x_2, --, x_n \geq 0$$

In this model, such variables in the objective function and in constraints are neutrosophic values as well as the second side of the relationships that represent constraints; this model is a nonlinear model if any component of the target function or constraints are nonlinear statements and the nonlinear statements may be in both.

4. Some Important Neutrosophic Definitions in Nonlinear Programming

a. The quadrature form of the neutrosophic target function:

A function with variables $f(x_1, x_2, --, x_n)$ is called a quadratic form if the following is true:

$$Nf(x_1, x_2, --, x_n) = \sum_{i=1}^{n}\sum_{j=1}^{n}(Nq_{ij})x_i x_j = X^T(NQ)\,X$$

Where $\boldsymbol{NQ_{n\times n}} = [\boldsymbol{Nq_{ij}}]$is a neutrosophic matrix, meaning that all or some of its elements are neutrosophic values of the shape $\boldsymbol{Nq_{ij}} = \boldsymbol{q_{ij}} + \boldsymbol{\varepsilon_{ij}}$ where$\boldsymbol{\varepsilon_{ij}}$ is the indeterminism of the elements of the matrix and any neighborhood may be close to $\boldsymbol{q_{ij}}$

It shall be written in one of the following forms: $[\boldsymbol{\lambda_{1ij}}, \boldsymbol{\lambda_{2ij}}]or\ \{\boldsymbol{\lambda_{1ij}}, \boldsymbol{\lambda_{2ij}}\}$or otherwise, and $X^T = (x_1, x_2, --, x_n)$

Here we can always assume without prejudice to the generality of the problem that NQ is symmetrical, we can replace the matrix NQ with the symmetrical matrix $\frac{(NQ+NQ^T)}{2}$ without changing the quadratic form.

b. Basic Minor Neutrosophic Matrices:

If the matrix NQ has dimensions$n \times n$, then the basic minimum matrix of the rank k is a mini-matrix with $k \times k$ dimensions and we obtain it by neglecting any $(n - k\ )$ lines and the corresponding columns of the matrix

Note: The determinant of the basic minor neutrosophic matrix is called the basic determinant and for each square matrix of the rank, $n \times n$ there is $2^n - 1$basic determinant.

c. Main Basic Minor Neutrosophic Matrices:

The main basic minor neutrosophic matrices from rank k for matrix $n \times n$ are obtained from the matrix NQ by neglecting the last lines $n - k$ and its corresponding columns.

Example 1: Find the basic minor matrices and the main basic minor matrices from the following neutrosophic matrix:

$$NQ = \begin{bmatrix} [0,1] & [2,3] & [4,5] \\ [6,7] & [8,9] & [10,11] \\ [12,13] & [14,15] & [16,17] \end{bmatrix}$$

Basic minor matrices:

Of the first rank: the elements of the main diameter [0,1][8,9] ,[16,17]

From the second rank: the following matrices:

$$\begin{bmatrix} [0,1] & [2,3] \\ [6,7] & [8,9] \end{bmatrix}, \begin{bmatrix} [6,7] & [10,11] \\ [12,13] & [16,17] \end{bmatrix}, \begin{bmatrix} [8,9] & [10,11] \\ [14,15] & [16,17] \end{bmatrix}$$

From the third rank: NQ

Main basic minor matrices:

From the first rank, [0,1](we neglect the last two lines and the corresponding columns).

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As for the main basic minor matrix of the second rank it is: $\begin{bmatrix} [0,1] & [2,3] \\ [6,7] & [8,9] \end{bmatrix}$

And the main basic minor matrix from the third rank is NQ.

Note: The number of major basic minor matrices in the matrix $n \times n$ the value n

d. Neutrosophic function gradation: $Nf(x_1, x_2, --, x_n)$

The neutrosophic function gradient is defined by the relation

$$\nabla f(x_1, x_2, --, x_n) = \left[\frac{\partial Nf}{\partial x_1}, \frac{\partial Nf}{\partial x_2}, ---, \frac{\partial Nf}{\partial x_n}\right]$$

e. Hessian matrix of the neutrosophic function $Nf(x_1, x_2, --, x_n)$**:**

It is a square and symmetrical matrix of the rank$n \times n$, denoted by $NH(x_1, x_2, --, x_n)$ and defined by the following relationship:

$$NH(x_1, x_2, --, x_n) = \left[\frac{\partial^2 Nf}{\partial Nx_i \partial Nx_j}\right]$$

f. Positive defined neutrosophic matrix:

A symmetrical neutrosophic matrix $NQ$is positively defined if the following is true**:**

- All diagonal elements must be positive.
- The main basic determinants should be positive.

Using the quadratic form, it is defined as follows:

We say about a matrix NQ that it is positive and defined if and only if the quadratic form satisfies the following: $X^T(NQ)X > 0$ For all X values

g. Positive quasi-defined neutrosophic matrix:

A symmetrical neutrosophic matrix is NQ a quasi-positive definition if the following is true**:**

- All diagonal elements must be non-negative.
- Basic determinants must be non-negative.

Using the quadratic form, it is defined as follows:

We say about a matrix NQ that it is positive quasi-defined if and only if the quadratic form satisfies the following: $(NQ)X \geq 0X$ for all X^Tvalues.

Note 1: To prove that a matrix NQ is negative (or negative quasi-defined) (NQ -) must be positive (or positive quasi-defined).

Using the quadratic form, it is defined as follows:

h. The defined neutrosophic matrix is negative:

We say that a matrix NQ is defined and negative if and only if $(-NQ)$is positive and defined or if the quadratic form has satisfied$(NQ)X < 0X$ for all X^T values

i. The quasi-defined neutrosophic matrix is negative:

We say about a matrix, NQ that it is quasi-negative and defined if and only if $(-NQ)$is positive and defined or if it satisfied the quadratic form: $X^T(NQ)X \leq 0$

Note 2: We say about Q matrix that it is not defined if $X^T(NQ)X$ was positive for values and negative for other values.

Note 3: The previous tests apply only to symmetrical matrices (and if the matrixNQ is asymmetrical, we replace it with the matrix $\frac{NQ-NQ^T}{2}$and apply the tests.

j. Convex function: The function Nf is a convex function if the Hessian matrix of this function is positive or quasi-positive and defined for all variables.

k. Concave Function: The function Nf is concave if the Hessian matrix of this function has negative or quasi-negative and defined for all variables.

Example: Show whether the functionNf is convex or concave, where $Nf$is given by the following relationship

$$Nf = [3,3.5]x_1^2 + 4x_2^2 + 3x_3^2 + 2x_1x_2 + 3x_1x_3 + x_2x_3 - 4x_1 + 5x_2 + 3x_3$$

We find the partial derivatives of the function s$Nf$with respect to the variable x_1, x_2, x_3

$$\frac{\partial Nf}{\partial x_1} = 2[3,3.5]x_1 + 2x_2 + 3x_3 - 4$$

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$$\frac{\partial Nf}{\partial x_2} = 8x_2 + 2x_1 + x_3 + 5$$
$$\frac{\partial Nf}{\partial x_3} = 6x_3 + 3x_1 + x_2 + 3$$

Then we find the Hessian matrix of the function Nf

$$H_{Nf}(x_1, x_2, --, x_n) = \left[\frac{\partial^2 Nf}{\partial Nx_i \partial Nx_j}\right] = \begin{bmatrix} \frac{\partial^2 Nf}{\partial x_1^2} & \frac{\partial^2 Nf}{\partial x_1 \partial x_2} & \frac{\partial^2 Nf}{\partial x_1 \partial x_3} \\ \frac{\partial^2 Nf}{\partial x_2 \partial x_1} & \frac{\partial^2 Nf}{\partial x_2^2} & \frac{\partial^2 Nf}{\partial x_2 \partial x_3} \\ \frac{\partial^2 Nf}{\partial x_3 \partial x_1} & \frac{\partial^2 Nf}{\partial x_3 \partial x_2} & \frac{\partial^2 Nf}{\partial x_3^2} \end{bmatrix} = \begin{bmatrix} [6,7] & 2 & 3 \\ 2 & 8 & 1 \\ 3 & 1 & 6 \end{bmatrix}$$

And from it

$$H_{Nf} = \begin{bmatrix} [6,7] & 2 & 3 \\ 2 & 8 & 1 \\ 3 & 1 & 6 \end{bmatrix}$$

We note that H_{Nf} is symmetrical matrix and all diagonal elements are positive and the basic minimum determinants are positive because the main determinant of the third rank is equal to

$$\begin{vmatrix} [6,7] & 2 & 3 \\ 2 & 8 & 1 \\ 3 & 1 & 6 \end{vmatrix} = [6,7](8 \times 6 - 1) - 2(2 \times 6 - 3) + 3(2 - 3 \times 8) = [198,245] > 0$$

The main determinant of the second rank

$$\begin{vmatrix} [6,7] & 2 \\ 2 & 8 \end{vmatrix} = [6,7] \times 8 - 4 =$$
$$[48,56] - 4 = [44,52] > 0$$

The main determinant of the first rank $|[6,7]| > 0$

From the above, we can see that the Hessian matrix of Nf the function is positive, that is, the function Nf is a convex function.

1. The maximum value and the minimum value of a bound nonlinear function:

From the previous study, we can determine whether the value of a target function in a nonlinear model is a minimum value or a maximum value as follows:

- The value is minimal if the target is a convex function, and the set of constraints forms a convex zone.
- The value is maximal if the target is a concave function, and the set of constraints forms a convex zone.

5. Conclusion and Results

In this research, we presented a study of neutrosophic nonlinear dependencies, and what is mentioned in this research can be used when searching for optimal values for neutrosophic nonlinear models, as there are many practical problems that lead to nonlinear models and we need to obtain optimal solutions to them that take into account all the conditions and fluctuations that face us during the workflow, which is provided to us by the science of neutrosophy through the neutrosophic values that we can use to build neutrosophic nonlinear models and the specified through the realistic study of the issues under study, and we focus on the need to prepare complementary research for this research through which we present the most important methods used to find the optimal solution for nonlinear models using the concepts of neutrosophic science.

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