



Using the Inverse Transformation Method to Generate Random Variables that follow the Neutrosophic Uniform Probability Distribution

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Abstract

When conducting the simulation process for any of the systems according to classical logic, we start by generating random numbers belonging to the regular probability distribution on the field $[0, 1]$ using one of the known methods, and then we convert these random numbers into random variables that follow the probability distribution that the system to be simulated works with, the simulation process that we perform it gives specific results that do not take into account the changes that may occur in the work environment of the system, to obtain more accurate results In a previous research, we prepared a study through which we reached random neutrosophic numbers follow the uniform distribution of the neutrosophic on the field with $[0, 1]$ no determination that can be enjoyed by the two parties of the field, one or both together, it may be in the form of a group or a field in another research, we converted these neutrosophic random numbers into neutrosophic random variables that follow the neutrosophic exponential distribution using the opposite conversion method that depends on the cumulative distribution function of the probability distribution by which the system to be simulated works, in this research we have used a method The opposite transformation to generate random variables that follow the neutrosophic uniform distribution and we have reached relationships through which we can convert the neutrosophic random numbers that follow the neutrosophic uniform distribution defined on the domain $[0, 1]$ with the indeterminacy enjoyed by each end of the field, one or the other, into random variables that follow the neutrosophic uniform distribution, b, a which is a classical uniform distribution whose medians are not precisely defined values, one or both may be cognitive in the form of a set or a domain, so that it takes into account all possible cases of mediators while maintaining the condition, and the $b, a; a < b$ method was illustrated through an applied example and we came up with neutrosophic random variables that follow the uniform distribution that give us more accurate simulation results when used due to the indeterminacy of neutrosophic values.

Keywords: Uniform distribution; Simulation; Cumulative distribution function; Random numbers; Neutrosophic random variables

1. Introduction:

In the field of scientific research, we face many systems that cannot be studied directly, due to the great difficulty that can face us when studying, and the high cost in addition to the fact that some systems cannot be studied directly, here comes the importance of the simulation process in all branches of science, as it depends on applying the study to systems similar to real systems and then dropping these results if they are suitable on the real system, the simulation process It depends on generating a series of random numbers subject to a uniform probability distribution on the domain $[0, 1]$, and then converting these numbers into random variables subject to the law of

probability distribution by which the system to be simulated works by using known conversion methods, in front of the revolution brought about by the science of neutrosophic in all fields of It was founded by the American philosopher and mathematician Florentin Smarandache in 1995 [1-7], and presented as a generalization of fuzzy logic, especially intuitive fuzzy logic, and as an extension of the fuzzy category theory presented by Lotfi A. Zadeh in 1965 [8]. It was necessary to keep pace with this development and formulate the basic concepts of the simulation process according to the basic concepts of neutrosophy, because the margin of freedom enjoyed by neutrosophic values gives more accurate results, which prompted many researchers to prepare many researches in various fields of science, especially in the field of mathematics and its applications [9-22], we present in this research a study aimed at obtaining neutrosophic random variables that follow the uniform neutrosophic distribution, which we will use when simulating systems that operate according to the uniform distribution in order to obtain more accurate simulation results by taking advantage of the indeterminacy enjoyed by neutrosophic values.

Discussion:

Since the aim of this paper is to obtain relationships in which we can convert neutrosophic random numbers that follow the neutrosophic uniform distribution on the field with the indeterminacy $[0,1]$ enjoyed by both sides of the field, one or the other, into neutrosophic random variables that follow the neutrosophic uniform distribution, which is a classical uniform distribution whose medians are not precisely defined b, a , one or both may be defined in doubt. For a group or domain, taking into account all possible cases of intermediaries and maintaining the condition $a < b$

2. Using classical values to generate trace random numbers for uniform distribution on the domain $[0,1]$: [23,24]

To generate a series of random numbers using the mean-squared method given by the following relationship: R_1, R_2, \dots

$$R_{i+1} = \text{Mid}[R_i^2]; i = 0, 1, 2, 3, \dots \quad (1)$$

Mid stands for the middle four ranks of R_i^2 and any R_0 random fractional number consisting of four orders (called a seed) and does not have a zero in any of its four ranks.

3. Converting random numbers that follow for uniform distribution over the domain $[0,1]$ to neutrosophic random numbers: [25]

To convert the resulting numbers from (1) to random neutrosophic numbers trace for uniform distribution on the domain $[0,1]$ we distinguish the following cases of the domain $[0,1]$ with a margin of indeterminacy .

The first case: the indeterminacy in the minimum field $[0 + \delta, 1]$ in this case we compensate by the δ where $\delta \in [0, m]$; $0 < m < 1$, and R_i are the random numbers resulting from the relationship (1), by the following relationship:

$$NR_i = \frac{R_i - \delta}{1 - \delta}$$

The second case: the indeterminacy in the maximum limit of the field in this case we replace

$[0, 1 + \delta]$ in this case we compensate by the δ where $\delta \in [0, m]$; $0 < m < 1$, and R_i are the random numbers resulting from the relationship (1), by the following relationship:

$$NR_i = \frac{R_i - \delta}{1 + \delta}$$

The third case: the indeterminacy in the upper and lower limits of the field in this case we replace $[0 + \delta, 1 + \delta]$ in this case we compensate by the δ where $\delta \in [0, m]$; $0 < m < 1$, and R_i are the random numbers resulting from the relationship (1), by the following relationship:

$$NR_i = R_i - \delta$$

1- finding the cumulative distribution function of the neutrosophic uniform distribution: [25]

to find the cumulative distribution function $F(x)$ of the neutrosophic uniform distribution given by the following

$$\text{density probability function: } f_N(x) = \frac{1}{b-a}$$

Where b, a is one or both of them not precisely defined, defined in the form of a group, domain, etc., we take into account all possible cases of intermediaries b, a while maintaining the condition $a < b$.

Accordingly, the cumulative distribution function takes one of the following forms, depending on the field of definition:

Figure I: Lack of Limitation in the Minimum Domain $[a_1 + \varepsilon, b]$

where and $a = a_N = a_1 + \varepsilon$; $\varepsilon \in [0, n]$; $a < n < b$ in this case the cumulative distribution function of the neutrosophic uniform distribution is given by the following relation

$$NF(x) = \frac{x - a_1}{b - a_N} - i; i \in \left[0, \frac{n}{b - a_N}\right]$$

Figure II: Indeterminacy at the upper limit of the domain $[a, b_1 + \varepsilon]$

where $b = b_N = b_1 + \varepsilon$; $\varepsilon \in [0, n]$; $a < n < b$ in this case, the cumulative distribution function of the neutrosophic uniform distribution is given by the following relationship:

$$NF(x) = \frac{x - a}{b_N - a}$$

Figure III: Indeterminacy in the upper and lower limits of the domain $[a_1 + \varepsilon, b_1 + \varepsilon]$ where $a = a_N = a_1 + \varepsilon$ and $b = b_N = b_1 + \varepsilon$ and $\varepsilon \in [0, n]$; $a < n < b$ this case the cumulative distribution function of the neutrosophic uniform distribution is given by the following relationship:

$$NF(x) = \frac{x - a_1}{b_N - a_N} - i; i \in \left[0, \frac{n}{b_N - a_N}\right]$$

Current study:

We want to generate random neutrosophic variables that follow the structured distribution and enact the opposite transformation method that is not defined according to the following rule:

$R = F(x) \Rightarrow x = F^{-1}(R)$ where $F(x)$ is the cumulative distribution function of the uniform distribution that follows the random variables, then if R_1, R_2, \dots it is a sequence of random numbers then the R_i probability is defined as follows:

$$f_R(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

and a cumulative distribution function:

$$F_R(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

From (1) and (2) the opposite conversion method to generate the random mutants that follow the neutrosophic uniform distribution distinguish three cases:

The first case: indeterminacy in the minimum field, i.e. $[a_1 + \varepsilon, b]$ where $a = a_1 + \varepsilon$; $\varepsilon \in [0, n]$; $a < n < b$ And we have: $a = a_N = a_1 + \varepsilon = a_1 + [0, n] = [a_1, a_1 + n]$

we use the relationship $R = F(x) \Rightarrow x = F^{-1}(R)$ we find

$$NF(x) = NR \Rightarrow NR = \frac{x - a_1}{b - a} - i \Rightarrow$$

$$\frac{x - a_1}{b - a} = NR + i \Rightarrow (NR + i)(b - a) = x - a_1 \Rightarrow$$

$$Nx = (NR + i)(b - a) + a_1$$

$$i \in \left[0, \frac{n}{b - a_N}\right]$$

Therefore, to generate random variables following the neutrosophic uniform distribution in the case of indeterminacy in the minimum domain we use the following relationship:

$$Nx_j = (NR_j + i)(b - a) + a_1 \quad ; \quad j = 0,1,2,3, \dots$$

The second case: the indeterminacy is related to the upper limit of the domain, i.e. $[a, b_1 + \varepsilon]$ where $\varepsilon \in [0, n]$, $a < n < b$ and we have $b = b_N = b_1 + \varepsilon = b_1 + [0, n] = [b_1, b_1 + n]$

Using the relationship $R = F(x) \Rightarrow x = F^{-1}(R)$ we find

$$NF(x) = \frac{x - a}{b_N - a}$$

$$NF(x) = NR \Rightarrow NR = \frac{x - a}{b_N - a} \Rightarrow$$

$$\frac{x - a}{b_N - a} = NR \Rightarrow NR(b_N - a) = x - a \Rightarrow$$

$$Nx = NR(b_N - a) + a$$

$$Nx = NR[(b_1 + \varepsilon) - a] + a$$

Therefore, to generate random variables that follow the neutrosophic uniform distribution in the case of indeterminacy related to the upper bound we use the following rule:

$$Nx_j = NR_j[(b_1 + \varepsilon) - a] + a \quad ; \quad j = 0,1,2,3, \dots$$

Third case : indeterminacy in the upper and lower limits of the domain $[a_1 + b, b_1 + \varepsilon]$ where $a = a_N = a_1 + b$, $b = b_N = b_1 + \varepsilon$, $\varepsilon \in [0, n]$; $a < n < b$

Using the relationship $R = F(x) \Rightarrow x = F^{-1}(R)$ we find

$$NF(x) = NR \Rightarrow NR = \frac{x - a_1}{b - a} - i \Rightarrow$$

$$\frac{x - a_1}{b - a} = NR + i \Rightarrow (NR + i)(b - a) = x - a_1 \Rightarrow$$

$$Nx = (NR + i)(b - a) + a_1$$

$$i \in \left[0, \frac{n}{b_N - a_N} \right]$$

Therefore, to generate random variables that follow the neutrosophic uniform distribution in the case of indeterminacy related to the lower and upper limits we use the following relationship:

$$Nx_j = (NR_j + i)(b - a) + a_1 \quad ; \quad j = 0,1,2,3,4$$

In the following, we will present an applied example that we study according to the classical logic and neutrosophic logic to be able to compare.

Practical example of classical values:

We assume that we have a system that works according to the regular distribution on the field $[3, 9]$ and we want to simulate the workflow of this system.

We apply the previous study:

- 1- We generate random numbers on the domain $[0, 1]$ using the average squared method, we take the seed we $R_0 = 0,2156$ get the following numbers:

$$R_0 = 0,2156 \quad , \quad R_1 = 0,6483 \quad , \quad R_2 = 0,0292 \quad , \quad R_3 = 0,0852 \quad , \quad R_4 = 0,7259$$

- 2- We know that the cumulative distribution function of the uniform distribution defined over the domain $[a, b]$ is given by the following relationship:

$$F(x) = \frac{x-a}{b-a}$$

Using the opposite conversion method, we obtain the following relationship:

$$R = F(x) = \frac{x-a}{b-a} \Rightarrow x = (b-a)R + a$$

Therefore, to convert the random numbers that follow the regular distribution on the field $[0, 1]$ into random variables that follow the regular distribution on the field, $[a, b]$ we substitute in the following relationship:

$$x_i = (b-a)R_i + a \quad ; \quad i = 0,1,2,3, \dots$$

Therefore, the random variables that follow the regular probability distribution on the domain $[3, 9]$ and corresponding to the following random numbers:

$$R_0 = 0,2156 \Rightarrow x_0 = (9-3) \times 0,2156 + 3 = 4.2936$$

$$R_1 = 0,6483 \Rightarrow x_1 = (9-3) \times 0,6483 + 3 = 6,8898$$

$$R_2 = 0,0292 \Rightarrow x_2 = (9-3) \times 0,0292 + 3 = 3.1752$$

$$R_3 = 0,0852 \Rightarrow x_3 = (9-3) \times 0,0852 + 3 = 3.5112$$

$$R_4 = 0,7259 \Rightarrow x_4 = (9-3) \times 0,7259 + 3 = 7.3554$$

Practical example neutrosophic values:

We suppose we have a system that works according to the uniform neutrosophic distribution on the field , where this means that it is $[3, 9]$ the indeterminacy $\varepsilon = [0, 3]$ that one or both ends of the field can take, $n = 3$ and we want to perform a neutrosophic simulation of the functioning of this system.

We apply the previous study:

- 3- We generate random numbers on the domain $[0, 1]$ using the average squared method, we take the seed we get the following numbers: $R_0 = 0,2156$

$$R_0 = 0,2156 \quad , \quad R_1 = 0,6483 \quad , \quad R_2 = 0,0292$$

$$R_3 = 0,0852 \quad , \quad R_4 = 0,7259$$

- 4- We convert these random numbers to neutrosophic random numbers we will apply the example to the third case when the indeterminacy is in the upper and lower limits of the field, $[0 + \delta, 1 + \delta]$ i.e. where $\delta = [0, 0.02]$ with the indication that we can obtain neutrosophic random numbers if the indeterminacy is only at the upper limit or at the lower limit only, using the appropriate relation for each case

We substitute in the appropriate relationship for the third case, which is: and $NR_i = R_i - \delta \quad \delta = [0, 0.02]$

$$NR_0 = 0,2156 - [0, 0,02] = [0.1956, 0.2156]$$

$$NR_1 = 0,6483 - [0, 0,02] = [0,6283, 0.6483]$$

$$NR_2 = 0,0292 - [0, 0,02] = [0.0092, 0,0292]$$

$$NR_3 = 0,0852 - [0, 0,02] = [0.0652, 0,852]$$

$$NR_4 = 0,7259 - [0, 0,02] = [0.7059, 0.7259]$$

We get the following neutrosophic random numbers:

$$NR_0 = [0.1956, 0.2156] \quad , \quad NR_1 = [0,6283, 0.6483]$$

$$NR_2 = [0.0092, 0,0292] \quad , \quad NR_3 = [0.0652, 0,852] \quad , \quad NR_4 = [0.7059, 0.7259]$$

To perform neutrosophic simulation we distinguish three cases:

The first case: the uniform distribution is classic, i.e., the probability density function and the neutrosophic

$$f(x) = \frac{1}{b-a}$$

random numbers

The second case: the uniform neutrosophic distribution given by the probability density function, i.e., one or both

$$f_N(x) = \frac{1}{b-a}$$

sides of the field

are not precisely defined, and random numbers are classic.
The third case: the uniform neutrosophic distribution and the neutrosophic random numbers We will apply the example to the third case, knowing that we follow the same method of solution for the first and second cases, using the appropriate laws for each case:

In the third case, the random numbers are neutrosophic, i.e., we take:

$$NR_0 = [0.1956, 0.2156] \quad , \quad NR_1 = [0,6283, 0.6483]$$

$$NR_2 = [0.0092, 0,0292] \quad , \quad NR_3 = [0.0652, 0,852] \quad , \quad NR_4 = [0.7059, 0.7259]$$

and uniform neutrosophic distribution with no limitation of both sides of the field, i.e. indeterminacy related to lower and upper limits

of the data we have $[a_1, b_1] = [3, 9]$ and $\varepsilon = [0, 3]$ and therefore

$$[a_N, b_N] = [a_1 + \varepsilon, b_1 + \varepsilon] = [3 + [0, 3], 9 + [0, 3]] = [[3, 6], [9, 12]]$$

Here to generate random variables following this distribution we use the following relationship:

$$Nx_j = (NR_j + i)(b - a) + a_1 \quad ; \quad j = 0,1,2,3,4$$

$$i \in \left[0, \frac{n}{b_N - a_N} \right]$$

First, we calculate and compensate $i = \frac{a_N - b_N}{b_N - a_N} = \frac{0 - 3}{12 - 0} = -0.25$

$$i = \left[0, \frac{a_N - b_N}{b_N - a_N} \right] = \left[0, \frac{0 - 3}{12 - 0} \right] = \left[0, -0.25 \right] = \left[0, 0.25 \right]$$

Substitute

$$Nx_j = ((NR_j + i)(b - a)) + a_1 = ((NR_j + [0, 0.25])(9 - 3)) + 3 \Rightarrow$$

$$Nx_j = ([0, 0.25NR_j] \times 6) + 3 = ([0, 3NR_j] + 3) \Rightarrow$$

$$Nx_j = [3, 3NR_j + 3]$$

$$NR_0 = [0.1956, 0.2156] \Rightarrow Nx_0 = [3, 3NR_0 + 3] = [3, 3[0.1956, 0.2156] + 3]$$

$$\Rightarrow Nx_0 = [3, [3.5868, 3.6468]]$$

$$NR_1 = [0.6283, 0.6483] \Rightarrow Nx_1 = [3, 3NR_1 + 3] = [3, 3[0.6283, 0.6483] + 3]$$

$$\Rightarrow Nx_1 = [3, [4.8849, 4.9449]]$$

$$NR_2 = [0.0092, 0.0292] \Rightarrow Nx_2 = [3, 3NR_2 + 3] = [3, 3[0.0092, 0.0292] + 3]$$

$$\Rightarrow Nx_2 = [3, [3.0276, 3.0876]]$$

$$NR_3 = [0.0652, 0.8520] \Rightarrow Nx_3 = [3, 3NR_3 + 3] = [3, 3[0.0652, 0.8520] + 3] \Rightarrow$$

$$Nx_3 = [3, [4.1956, 5.5560]]$$

$$NR_4 = [0.7059, 0.7259] \Rightarrow Nx_4 = [3, 3NR_4 + 3] = [3, 3[0.7059, 0.7259] + 3]$$

$$\Rightarrow Nx_4 = [3, [5.1177, 5.1777]]$$

4. Conclusion and results:

We note that the values corresponding to the random variables that we obtained when using the classical values are specific values and they express states of the studied system, which does not correspond to the reality of the situation and the working conditions of the system in light of the momentary changes and fluctuations that it can go through.

But the corresponding values of the variables obtained by using neutrosophic values are field values that suit the environment and working conditions of the system that works according to the uniform distribution defined in the example, and therefore working using the concepts of neutrosophic science gives more accurate results due to the margin of freedom that we obtain through the indeterminacy provided by the science of neutrosophy. In the near future, we look forward to preparing a study through which we can generate random variables that follow other probability distributions by converting random numbers that follow the regular distribution into random variables that follow the appropriate probability distribution for the case under study.

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