Optimal Value of the Service Rate in the Unlimited Model $M\backslash M\backslash 1$

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Abstract

Operations research is the applied side of mathematics, and since its inception it has helped to improve the performance of many systems that used its methods in its workflow, one of the methods of operations research is the theory of queues that have been used in many aspects of life, especially the aspects that are directly related to customer service, and the goal is to serve the customer as soon as possible and at the lowest cost, which prompted many researchers to provide the research and studies that can be applied in systems that have a queue.These studies showed good results, from these studies this research is presented to find the optimal value of the service rate in the unlimited model $M\backslash M\backslash 1$ according to classical logic, we present in this research a study to find the relationship through which we get the optimal value of the service rate using neutrosophic logic and this study is an expansion of the study according to classical logic, through the two studies The great benefit we get when using neutrosophic logic and neutrosophic values in operations research topics is demonstrated.

Keywords: Operations Research; Neutrosophic Logic; Queues– $M\backslash M\backslash 1$ Model; Service Rate; Neutrosophic Service Rate-Optimum Value

Introduction

The search for optimization is the goal of all studies offered by operations research methods, and keeping pace with scientific development is necessary to obtain results that guarantee us a safe workflow, so many topics of operations research have been studied.

Using neutrosophic logic, such as fuzzy linear models, Static inventory models, Decision Theory, The process of converting neutrosophic random numbers into random variables tracking for exponential distribution- Artificial intelligence and neutrosophic machine learning in the diagnosis and detection of Covid-19 -Study of a support vector machine algorithm with a perpendicular logander core and reverse interpolation of Lagrange according to the neutrosophic logic [1-9] and the aim was to obtain more accurate results that ensure a secure workflow for the systems that use it, in order to provide optimal service and reduce customer waiting time in any of the service centers, researchers in the field of operations research presented the theory of neutrosophic queues and the basic indicators were reformulated using the concepts of neutrosophic logic. [10,11] Complement to previous research, we present in this research a study through which we determine the optimal value of the customer service rate according to the neutrosophic logic, because when the matter is related to the services provided to customers, we need values that give us a margin of freedom through which to get out of the restriction and embarrassment that we can fall into when the circumstances surrounding the work environment prevent the implementation of the set plan, this is what makes us agree that when replacing the classical values with neutrosophic values, we switch the value $T(\mu)$ that express the cost of the service with a group that we denote by $T(\mu_N)$ and by which we mean (neutrosophic $T(\mu)$) or (inaccurate $T(\mu)$) or (unspecified $T(\mu)$), which may be adjacent to or may be a period containing $T(\mu)$ depending on what is mentioned in reference No. [12], in general it can be considered any group close to $T(\mu)$ that enables us to meet the customers requirements in the shortest time and lowest cost.

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Discussion:

The process of searching for the optimal solution requires building a mathematical model that expresses the situation under study and then searching for the optimal solution according to the methods of the type of this model, the type of service system in the queues greatly affects the volume of services provided and any error in the organization of these services exacerbates the problem, in the model the average number of customers or requests that the system can serve during one time, which is called the speed of the service or the ability to pass the system is one of the basic indicators in the model $M\backslash M\backslash \mathbf{1}$ [13,14] and when we can get an optimal value for it we guarantee a good functioning of the system with the lowest roll. This value is determined by building a mathematical model and solving this model is the required value in this research, we present two studies, the first is to address the problem according to classical logic and the second is to address the problem according to neutrosophic logic.

First: Formulating a problem according to classical logic based on what is mentioned in the references [13,14]:

A customer service center operates according to the unspecified model $M\backslash M\backslash 1$, the average number of customers or orders submitted during the unit of time λ, the average number of customers or orders that can be served during unit of time μ, the cost of serving one customer in the center, the cost of λμ waiting for the customer in the center during the unit of timeC_1 , we denote the value of the total cost of waiting and service by code $T(\mu)$, which is given by the following relationship:

$$T(\mu) = C_1\mu + C_2L_s \qquad (1)$$

Where $\boldsymbol{L_s}$ is the average number of people inside the system (in the queue and in the service center) and is given by the following relationship:

$$\boldsymbol{L_s = \frac{\lambda}{\mu - \lambda}}$$

We substitute in relation (1)

$$T(\mu) = C_1\mu + C_2\,\frac{\boldsymbol{\lambda}}{\boldsymbol{\mu - \lambda}}$$

We get the function of total waiting cost

The mathematical model is as follows:

Find the minimum value of

$$T(\mu) = C_1\mu + C_2\,\frac{\boldsymbol{\lambda}}{\boldsymbol{\mu - \lambda}} \longrightarrow \boldsymbol{Min}$$

Within the conditions

$$\lambda < \mu$$

$$\mu\,, \lambda \geq 0$$

We note that the function is a nonlinear function and with one variable is μ to find the optimal value we derive this function with respect to we find:

$$T'(\mu) = C_1 + C_2\,\frac{\boldsymbol{-\lambda}}{\boldsymbol{(\mu - \lambda)^2}}$$

$$C_1 + C_2\,\frac{\boldsymbol{-\lambda}}{\boldsymbol{(\mu - \lambda)^2}} = 0 \Rightarrow \mu = \lambda + \sqrt{\frac{C_2.\lambda}{C_1}}$$

$$\mu^* = \lambda + \sqrt{\frac{C_2.\lambda}{C_1}} \qquad (*)$$

Through the relationship $(*)$ we can find the classical optimal value of the service rate in the model $M\backslash M\backslash 1$

Second: Formulating the issue according to the logic of neutrosophic based on what is mentioned in the article [12,13]:

The neutrosophic function that has some indefinite (indeterminacy), taking into account the definition of its field and its corresponding field, and therefore the previous mathematical model becomes a

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neutrosophic mathematical model if the goal function is a neutrosophic function by looking at the variables and constants in this function, we find that the model turns into a neutrosophic model if we take instead of the following values : μ, λ, C_1, C_2:

Neutrosophic values i.e., $\mu_N, \lambda_N, C_{1N}, C_{2N}$, which may be adjacent to or may be a period containing μ, λ, C_1, C_2 depending on what is mentioned in reference No. [12], in general it can be considered any group close to μ, λ, C_1, C_2 that enables us to meet the customers requirements in the shortest time and lowest cost

A customer service center operates according to the unspecified model $NM\backslash NM\backslash 1$, the average number of customers or orders submitted during the unit of time λ_N, the average number of customers or orders that can be served during the unit of time μ_N, the cost of serving one customer in the center C_{1N}, the cost waiting for the customer in the system during the unit of time C_{2N}, we denote the value of the total cost of waiting and service by code $T(\mu_N)$, which is given by the following relationship:

$$T(\mu_N) = C_{1N}\mu_N + C_{2N}L_{sN} \qquad (1)$$

Where L_{sN} is the average number of people inside the system (in the queue and in the service center) and is given by the following relationship:

$$L_{sN} = \frac{\lambda_N}{\mu_N - \lambda_N}$$

We substitute in relation (1)

$$T(\mu_N) = C_{1\mathrm{N}}\mu_N + C_{2\mathrm{N}}\frac{\lambda_N}{\mu_N - \lambda_N}$$

We get the total cost of waiting function

The mathematical model is as follows:

Find the minimum value of

$$T(\mu_N) = C_{1\mathrm{N}}\mu_N + C_{2\mathrm{N}}\frac{\lambda_N}{\mu_N - \lambda_N} \longrightarrow Min$$

Within the conditions

$$\lambda_N < \mu_N$$
$$\mu_N, \lambda_N \geq 0$$

We note that the function is a nonlinear function and with one variable which is μ_N to find the optimal value we derive this function with respect to μ_N we find:

$$T'(\mu_N) = C_{1\mathrm{N}} + C_{2\,\mathrm{N}}\frac{-\lambda_N}{(\mu_N - \lambda_N)^2}$$

$$T'^{(\mu_N)} = C_{1\mathrm{N}} + C_{2\,\mathrm{N}}\frac{-\lambda_N}{(\mu_N - \lambda_N)^2} = 0 \Rightarrow \mu_N = \lambda_N + \sqrt{\frac{C_{2N}.\lambda_N}{C_{1N}}}$$

$$\mu_N^* = \lambda_N + \sqrt{\frac{C_{2N}.\lambda_N}{C_{1N}}} \qquad (**)$$

Through the relationship $(**)$ we can find the optimal neutrosophic value for the service rate in the model $NM\backslash NM\backslash 1$

It should be noted that it is not necessary that all values μ, λ, C_1, C_2 are neutrosophic values in order to get an optimal neutrosophic value for the service rate, it is enough to have one neutrosophic value in the target function.

How to benefit from the previous study [10]:

It is noted that the purpose of studying neutrosophic queues is to calculate a number of important indicators such as

1- Average customer waiting time inside the system (in queue and in service center)

$$NW_s = \frac{1}{\mu_N - \lambda_N}$$

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2- Average customer waiting time in queue only

$$NW_q = \frac{\lambda_N}{\mu_N(\mu_N - \lambda_N)}$$

NW_s is associated with NW_q by the relationship:

$$NW_s = \mathrm{NW}_q + \frac{1}{\mu_N}$$

3- Average number of customers inside the system (in queue and in the service center)

$$NL_s = \lambda_N . NW_s = \frac{\lambda_N}{\mu_N - \lambda_N}$$

4- Average number of customers in queue only

$$NL_q = \lambda_N . NW_q = \frac{{\lambda_N}^2}{\mu_N(\mu_N - \lambda_N)}$$

NL_s is associated with NL_q by the relationship:

$$NL_s = \mathrm{NL}_q + \frac{\lambda_N}{\mu_N}$$

5- The service rate is given by the following relationship:

$$\rho_N = \frac{\lambda_N}{\mu_N}$$

We note that these indicators are calculated after knowing λ_N and μ_N therefore in any service center, if we have the number of requests that this center accepts during the unit of time and we know the cost of serving one customer and also the cost of waiting for him in the system and we need to calculate the previous indicators, we need to calculate the optimal value of the service rate first and calculate it from the relationship $(**)$

Then we use the resulting value to calculate the required indicators , we explain the above through the following example:

Example 1:

Customers arrive at a gas station at rate $\lambda_N = \{3,4\}$ of one customer per hour find how long the customer stays in the system if you know that the cost of customer service is $C_1 = 100$ monetary unit and the cost of waiting in the system is $C_2 = 10$ a monetary unit

The average waiting of the customer in the system is calculated from the relationship

$$NW_s = \frac{1}{\mu_N - \lambda_N}$$

This means that we need to calculate the value of μ_N which is calculated from the relationship $(**)$ because the rate of arrival of customers to the station is a neutrosophic value $\lambda_N = \{3,4\}$, and therefore we find

$$\mu_N^* = \lambda_N + \sqrt{\frac{C_{2N} . \lambda_N}{C_{1N}}} = \{3,4\} + \sqrt{\frac{10 . \{3,4\}}{100}}$$

For $\lambda = 3$ we find

$$\mu^* = \lambda + \sqrt{\frac{C_2 . \lambda}{C_1}}$$

$$\mu^* = \lambda + \sqrt{\frac{C_2 . \lambda}{C_1}} = 3 + \sqrt{\frac{10 \times 3}{100}} = 3 + \sqrt{0.3} = 3.5$$

For $\lambda = 4$ we find

$$\mu^* = \lambda + \sqrt{\frac{C_2 . \lambda}{C_1}} = 4 + \sqrt{\frac{10 \times 4}{100}} = 4 + \sqrt{0.4} = 4.6$$

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Accordingly, it is a neutrosophic value.$\mu_N^* = \{3.5\ ,4.6\}$

We calculate the average length of customer waiting time in the system

$$NW_s = \frac{1}{\mu_N - \lambda_N} = \frac{1}{\{3.5\ ,4.6\} - \{3,4\}}$$

$$\mu = 3.5\ ,\lambda = 3 \Rightarrow W_s = \frac{1}{3.5 - 3} = 2$$

$$\mu = 3.5\ ,\lambda = 4 \Rightarrow W_s = \frac{1}{3.5 - 4}$$

This value is rejected because it is negative

$$\mu = 4.6\ ,\lambda = 3 \Rightarrow W_s = \frac{1}{4.6 - 3} = 0.625$$

$$\mu = 4.6\ ,\lambda = 4 \Rightarrow W_s = \frac{1}{4.6 - 4} = 1.7$$

Therefore,$NW_s = \{2\ ,0.6\ ,1.7\}$ and it is a neutrosophic value.

Example 2:

Car repair shop receives customers at an hourly rate$\lambda_N = [5,10]$.

Find the optimal value of the service rate if you know that the cost of customer service is $C_{1N} = [15,20]$ dollars and the cost of waiting in the system is $C_{2N} = [3,8]$ dollars.

You will be able to find the optimal neutrosophic value of the service rate through the following relationship:

$$\mu_N^* = \lambda_N + \sqrt{\frac{C_{2N}.\lambda_N}{C_{1N}}}$$

$$\mu_N^* = \lambda_N + \sqrt{\frac{C_{2N}.\lambda_N}{C_{1N}}} = [5,10] + \sqrt{\frac{[3,8].[5,10]}{[15,20]}} = [5,10] + \sqrt{\frac{[15,80]}{[15,20]}} = [5,10] + [1,2]$$

$$\mu_N^* = [5,10] + [1,2] = [7,12]$$

After obtaining the value of μ_N^* and we know in the text of the problem the value of λ_N we can calculate all the indicators

Conclusion and Results:

Through the previous study, we note that the optimal value of the rate of service provided by the classic model solution is a specific value, and the validity of this value depends on the conditions surrounding the environment in which the system operates, but the optimal value that we obtain through the neutrosophic model solution is an indefinite value that takes into account the conditions surrounding the work environment because the values that enter into the optimal solution phrase (the average number of customers or orders submitted during the unit of time λ_N and the cost of serving one customer in the center C_{1N} and the cost of waiting for the customer in the system during the unit of time C_{2N} (neutrosophic value (some or all) are suitable for the worst conditions and the best for the system working environment.

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