



The State of Risk and Optimum Decision According to Neutrosophic Rules

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Abstract

In light of the current circumstances and the momentary fluctuations that we live in all domains of life, which have transformed most decision-making cases into risky cases, this study was necessary. In this research, we present a new study for decision-making in the event of risk, by formulating some rules according to the neutrosophic logic starting from the data of the problem, where we replaced the classical values in the matrix of profit and probabilities with neutrosophic values that have a margin of freedom thanks to the indeterminacy provided by the neutrosophic logic, and we got more accurate solutions than the solutions that we obtained when using the classical rules which reduces the occurrence of losses, especially if the decisions are fateful and the decision issues are huge and complex and helps us to achieve the greatest profits by adopting a rational decision based on modern quantitative ways and methods.

Keywords: Decision-Making Theory; Neutrosophic Logic; Risk Condition; Aspiration Level; Most Probable Case; Largest Expected Values.

1. Introduction:

In the world of rapid changes, there has been an urgent need to make a rational decision based on quantitative methods that reduce the risk ratio. The case of risk has been studied according to the classical logic and through the application of appropriate rules that enable us to reach the optimal decision using the amounts corresponding to the alternatives and states of nature.

Since these quantities are classical values, the decision will not be accurate enough to guarantee the greatest profits and the least losses.

As in all the topics of operations research and other fields of life, and after the great revolution brought about by the neutrosophic logic in all fields of science, where many researchers tended to reformulate many topics in different fields of science according to the basic concepts of this logic and many researches were published on operations research and most mathematics topics [1_17], In this research, we presented a study for decision-making in the case of risk, and it is an extension of the studies presented by many researchers using neutrosophic logic [18,19], which gave more accurate results, and helped the decision maker to take a rational decision that reduces risk because it is based on accurate scientific foundations.

2. Discussion:

the movement of nature is random and subject to a certain probability distribution, noting that the probability distribution may not be completely known, but rather a distribution assumed by experts, or by the decision maker himself, here is the risk that may be reflected in the decision itself to reduce them, the possibilities corresponding to the states of nature must be calculated, or estimated through practical facts or a statistical study taken from previous experiences and studies, so the researchers provided some rules using the concepts of classical logic

that can be relied upon during decision-making in the case of risk, by studying decision-making theory according to the classical logic as mentioned in the following references[20,21].

1- The issue of making a decision in case of risk according to the classical logic:

The decision maker has alternatives $A(a_1, a_2, \dots, a_m)$ where m the number of alternatives available to the decision maker and the situations that nature can take in the future $\theta(\theta_1, \theta_2, \dots, \theta_n)$ where n is the number of states that nature can take when in motion, and the amount of profit or loss that the decision maker will achieve is $X(a_i, \theta_j)$ or by short code X_{ij} . Then the profit matrix is given in the following table:

Table 1: profit matrix table

States of nature Alternatives	θ_1	θ_2	θ_3	----	θ_n
a_1	x_{11}	x_{12}	x_{13}	----	x_{1n}
a_2	x_{21}	x_{22}	x_{23}	----	x_{2n}
a_3	x_{31}	x_{32}	x_{33}	----	x_{3n}
-----	-----	-----	-----	-----	-----
a_m	x_{m1}	x_{m2}	x_{m3}	-----	x_{mn}

In addition to the law of the probability distribution that governs the possible states of nature which is a numerical series or a mathematical function corresponding to each state of nature, as follows: $P(P_1, P_2, \dots, P_n)$ where $P_j \geq 0$ & $\sum_{j=1}^n P_j = 1$ and the goal of the decision maker is to obtain the greatest possible profit or the least possible loss. Then it is summarized in the following table:

Table 2: Profit table and odds

States of nature Alternative	θ_1	θ_2	θ_3	----	θ_n
a_1	x_{11}	x_{12}	x_{13}	----	x_{1n}
a_2	x_{21}	x_{22}	x_{23}	----	x_{2n}
a_3	x_{31}	x_{32}	x_{33}	----	x_{3n}
-----	-----	-----	-----	-----	-----
a_m	x_{m1}	x_{m2}	x_{m3}	----	x_{mn}
P_j	P_1	P_2	P_3	----	P_n

To determine the optimal decision, the classic study presented rules, including as mentioned in

References[20, 21]:

1-1- Ambition Level Rule:

In this rule the decision maker sets a certain level of profit and another level of loss and makes the decision to achieve both levels.

Practical example (1-1) We have the following table of alternatives and states of nature:

Table 3: profit matrix table

States of nature Alternative	θ_1	θ_2	θ_3
a_1	300	100	400

a_2	-220	170	500
a_3	-400	200	300
a_4	160	250	200

It is required to determine the appropriate alternative using the level of ambition rule according to the following data: the level of ambition of the decision maker:

- The profit should not be less than 300 .
- That the loss does not exceed 200 .

By studying the elements of the table, we notice that the alternative line a_1 achieves the level of ambition of the decision-maker in profit.

And from the study of the alternative line a_2 , we note that it achieves the level of ambition of the decision-maker in profit as well as the alternative line a_3 , we note that it achieves the level of ambition of the decision maker in profit.

As for the alternative line a_4 , it does not achieve the level of ambition of the decision-maker in profit, so we exclude it.

Thus, we get the following alternatives, a_1, a_2, a_3 to achieve the level of ambition of the decision maker in profit.

By studying the elements of alternative a_2 , we note that it does not achieve the level of ambition of the decision-maker in the loss because of the presence of (-220) , so we exclude it.

By studying the elements of the a_3 alternative, we note that it does not achieve the level of ambition of the decision maker in the loss because of the presence of (-400) , so we exclude it

Thus, the appropriate alternative that achieves the decision-maker's ambition level in profit and loss is alternative a_1 and the greatest profit is 400 .

1-2 -Most Probable Case Rule (Modal): This state depends on the probability distribution of the movement of states of nature. We symbolize the corresponding probabilities of states of nature $(\theta_1, \theta_2, \dots, \theta_n)$ with symbols (P_1, P_2, \dots, P_n) , These possibilities must fulfill two conditions:

$$0 \leq P_j \leq 1 \quad \& \quad \sum_{j=1}^n P_j = 1$$

To achieve the rule we define PS where:

$$P_s = \text{Max}_j [P_j]$$

Then the natural state is θ_s it is the most frequent case that we limit ourselves to when calculating, and here the issue comes to a decision in case of confirmation.

We choose the alternative that achieves the largest profit from the column θ_s the corresponding decision is the appropriate decision.

Practical Example (1-2)

We have the following table of alternatives and states of nature and the corresponding probabilities of states of nature:

Table 4: Profit and Probability Matrix Table

States of nature Alternative	θ_1	θ_2	θ_3
a_1	300	100	400
a_2	-220	170	500
a_3	-400	200	300
a_4	160	250	200
P_j	0.3	0.1	0.6

Through the table, we note that the most probable case is case θ_3 then the issue will lead to a decision in the event of confirmation according to the following table:

Table 5: Most probable case table

Alternative	Case θ_3
a_1	400
a_2	500
a_3	300
a_4	200
P_j	0.6

We select the largest value in the status column θ_3 it is 500 interviews for the alternative a_2 and a_2 is the appropriate alternative

1-3 The Rule of the Largest Expected Values:

This rule depends on the following law:

$$E(a_k) = \text{Max}_i [E(a_i)]$$

Where $E(a_k)$ is the largest expected value and $E(a_i)$ is the expected value of the profit at the alternative and a_i is calculated from the relationship:

$$E(a_i) = \sum_{j=1}^n P_j x_{ij}$$

Then the alternative a_k is the appropriate alternative.

Practical Example (1-3)

We have the following table of alternatives and states of nature and the probabilities corresponding to states of nature:

Table 6: profit matrix table

States of nature Alternative	θ_1	θ_2	θ_3
a_1	300	100	400
a_2	-220	170	500
a_3	-400	200	300
a_4	160	250	200
P_j	0.3	0.1	0.6

We calculate the expected values for each of the alternatives, we get the following table:

Table 7: Table of expected values

States of nature Alternative	θ_1	θ_2	θ_3	E(ai)
a_1	300	100	400	$E(a_1) = 0.3 * 300 + 0.1 * 100 + 0.6 * 40 = 340$
a_2	-220	170	500	$E(a_2) = 0.3 * (-220) + 0.1 * 170 + 0.6 * 5 = 251$
a_3	-400	200	300	$E(a_3) = 0.3 * (-400) + 0.1 * 200 + 0.6 * 30 = 80$
a_4	160	250	200	$E(a_4) = 0.3 * 160 + 0.1 * 250 + 0.6 * 200 = 193$
P_j	0.3	0.1	0.6	

From the table, we notice that:

$$E(a_k) = \text{Max}_i [E(a_i)] = \text{Max}\{340, 251, 80, 193\} = 340 \Rightarrow E(a_k) = E(a_1)$$

Therefore, the appropriate alternative is a_1 and it is the appropriate decision.

In order to obtain a more accurate decision that helps us reduce the amount of risk, we have formulated in this research the decision-making Issue and the previous rules using the concepts of neutrosophic logic:

2- The Text of the Decision-Making Issue in the Case of Neutrosophic Risk:

At the outset, we note that the neutrosophic decision tree has been studied in light of the neutrosophic possibilities in the research [18].

In this research we will formulate the issue of decision-making in case of risk according to neutrosophic logic we get:

The decision maker has alternatives $A(a_1, a_2, \dots, a_m)$ where m is the number of alternatives available to the decision maker and the cases that nature can take in the future are $\theta(\theta_1, \theta_2, \dots, \theta_n)$ where n is the number of states that nature can take (They are independent of each other) when in motion and the amount of profit or loss that the decision maker will achieve, we symbolize it with the symbol $NX_{ij} = X(a_i, \theta_j) \pm \varepsilon_{ij}$ or with the short symbol NX_{ij} and they are neutrosophic values and ε_{ij} is indefinite, it could be $\varepsilon_{ij} = [\lambda_1, \lambda_2]$ or $\varepsilon_{ij} = \{\lambda_1, \lambda_2\}$ or ----- otherwise, also the law of the probability distribution to which the possible states of nature are subject, we take it as a neutrosophic numerical series or a neutrosophic mathematical function corresponding to each state of nature with its probability of occurrence:

$NP(\theta) = (NP(\theta_1), NP(\theta_2), NP(\theta_3), \dots, NP(\theta_n))$ and $-0 \leq \sum_{j=1}^n NP(\theta_j) \leq 3^+$ where $NP(\theta_j) = P(\theta_j) + i_j$ and i_j is indeterminate and we take it $i_j[\delta_{i1}, \delta_{i2}]$ or $i_j = \{\delta_{i1}, \delta_{i2}\}$ or ----- otherwise, within the previous data, the goal is to obtain the greatest possible profit or the lowest possible loss.

Organize the previous information in the following table:

Table No. 8: Table of Profit and neutrosophic probabilities

States of nature Alternatives	θ_1	θ_2	θ_3	----	θ_n
a_1	$x_{11} + \varepsilon_1$	$x_{12} + \varepsilon_1$	$x_{13} + \varepsilon_1$	----	$x_{1n} + \varepsilon_1$
a_2	$x_{21} + \varepsilon_2$	$x_{22} + \varepsilon_2$	$x_{23} + \varepsilon_2$	----	$x_{2n} + \varepsilon_2$
a_3	$x_{31} + \varepsilon_3$	$x_{32} + \varepsilon_3$	$x_{33} + \varepsilon_3$	----	$x_{3n} + \varepsilon_3$
-----	-----	-----	-----	-----	-----
a_m	$x_{m1} + \varepsilon_m$	$x_{m2} + \varepsilon_m$	$x_{m3} + \varepsilon_m$	----	$x_{mn} + \varepsilon_m$
$NP(\theta_j)$	$NP(\theta_1)$	$NP(\theta_2)$	$NP(\theta_3)$	----	$NP(\theta_n)$

To determine the optimal decision according to the neutrosophic logic, we reformulate the rules mentioned in the classical study according to the new data

2-1 Neutrosophic Aspiration Level Rule:

In this rule, the decision maker sets a certain level of profit, for example M and another level of loss, for example N and he studies the available alternatives and excludes every alternative that does not achieve his level of ambition then he determines the best alternative among the remaining alternatives by using one of the rules used in the case of uncertain data studied according to the neutrosophic logic in a previous research [Search I make the decision].

Practical Example (2-1)

We have the following table of alternatives and states of nature:

To be able to clarify the rule, we will take the same value of the indefinite for all alternatives lit it be $\varepsilon = [0,50]$, and we will get the following matrix of payments:

Table 9: Neutrosophic Profit Matrix Table

States of nature Alternatives	θ_1	θ_2	θ_3
a_1	[300,350]	[100,150]	[400,450]
a_2	[-220, -170]	[170,220]	[500,550]
a_3	[-400, -350]	[200,250]	[300,350]
a_4	[160,210]	[250,300]	[200,250]

It is required to determine the appropriate alternative using the level of ambition rule according to the following data: the level of ambition of the decision maker:

- a. That the profit belongs to the domain of $M = [300,350]$.
- b. That the loss belongs to the domain of $N = [200,250]$.

By studying the elements of the table, we notice that the alternative line a_1 achieves the level of ambition of the decision-maker in profit and from the study of the alternative line a_2 , we note that it achieves the level of ambition of the decision-maker in profit.

As well as the alternative line a_3 we note that it achieves the level of ambition of the decision maker in profit.

As for the alternative line a_4 , it does not achieve the level of ambition of the decision-maker in profit, so we exclude it.

Thus, we get the following alternatives, a_1, a_2, a_3 to achieve the level of ambition of the decision maker in profit.

By studying the elements of alternative a_2 , we note that it does not achieve the level of ambition of the decision-maker in losing because there is $[-220, -170]$ therefore we exclude it.

By studying the elements of the a_3 alternative, we note that it does not achieve the level of ambition of the decision-maker in losing because there is $[-400, -350]$ therefore we exclude it.

Thus, the appropriate alternative that achieves the level of ambition of the decision-maker in profit and loss is alternative a_1 .

2-2 Neutrosophic Most Probable Case Rule:

This state depends on the probability distribution of the movement of states of nature $\theta(\theta_1, \theta_2, \dots, \theta_n)$ we symbolize the probabilities corresponding to the states of nature with symbols $NP(\theta) = (NP(\theta_1), NP(\theta_2), NP(\theta_3), \dots, NP(\theta_n))$ which should check:

$$-0 \leq \sum_{j=1}^n NP(\theta_j) \leq 3^+$$

We specify $NP(\theta_s)$ where:

$$NP(\theta_s) = \max_j [NP(\theta_j)] ; j = 1, 2, 3, \dots, n$$

Then the state of nature θ_s is the most frequent state that we restrict to when calculating, here, the issue is to make a decision in the event of certainty. We choose the alternative that achieves the greatest profit from the column θ_s and the corresponding decision is the appropriate decision.

Practical Example (2-2)

We have the following table of alternatives and states of nature:

We take the same value of the indeterminacy for all alternatives, let it be $\epsilon = [0,50]$ and we take the indefinite $i_1, i_2, i_3 = [0,15]$, we get the following matrix of payments:

Table No. 10: Table of the matrix of profit and neutrosophic probabilities

States of nature Alternatives	θ_1	θ_2	θ_3
a_1	[300,350]	[100,150]	[400,450]
a_2	[-220, -170]	[170,220]	[500,550]
a_3	[-400, -350]	[200,250]	[300,350]
a_4	[160,210]	[250,300]	[200,250]
$P(\theta_j)$	[0.3,0.45]	[0.1,0.25]	[0.6,0.75]

It is required to determine the appropriate alternative using the most probable rule.

Through the table, we note that the most probable case is the case θ_2 in which the issue will lead to a decision in the event of confirmation according to the following table:

Table 11: Table of the most probable Neutrosophic state

States of nature Alternatives	θ_3
a_1	[400,450]
a_2	[500,550]

a_3	[300,350]
a_4	[250,300]
$P(\theta_j)$	[0.6,0.75]

We choose the largest value in the condition column θ_3 that [500,550] corresponds to the alternative a_2 and a_2 is the appropriate alternative.

2-3 The Rule of the Largest Expected Neutrosophic Values:

- a. We calculate $E(a_i)$ the expected value of the profit at each alternative a_i where $i = 1, 2, 3, \dots, m$ from the relation:

$$NE(a_i) = \sum_{j=1}^n NP(\theta_j) \cdot NX_{ij} \quad ; \quad i = 1, 2, 3, \dots, m$$

- b. We find $NE(a_k)$ which is the largest expected value, that is:

$$NE(a_k) = \text{Max}_i [NE(a_i)]$$

Then, the alternative a_k is the optimal alternative according to this rule.

Example (2-3)

We take the table in the previous case and form a table of expected values. We get the following table:

Table 12: Table of Neutrosophic Expected Values

States of nature Alternatives	θ_1	θ_2	θ_3
a_1	[300,350]	[100,150]	[400,450]
a_2	[-220, -170]	[170,220]	[500,550]
a_3	[-400, -350]	[200,250]	[300,350]
a_4	[160,210]	[250,300]	[200,250]
$NP(\theta_j)$	[0.3,0.45]	[0.1,0.25]	[0.6,0.75]

Applying the rule for all alternatives, we get the following table:

Table No. 13: Table of Expected Values

States of nature Alternatives	θ_1	θ_2	θ_3	$NE(a_i)$
a_1	[300,350]	[100,150]	[400,450]	$NE(a_1) = [0.3,0.45] * [300,350] + [0.1,0.25] * [100,150] + [0.6,0.75] * [400,450] = [340,532.5]$
a_2	[-220, -170]	[170,220]	[500,550]	$NE(a_2) = [0.3,0.45] * [-220, -170] + [0.1,0.25] * [170,220] + [0.6,0.75] * [500,550] = [251,391]$
a_3	[-400, -350]	[200,250]	[300,350]	$NE(a_3) = [0.3,0.45] * [-400, -350] + [0.1,0.25] * [200,250] + [0.6,0.75] * [300,350] = [80,167.5]$
a_4	[160,210]	[250,300]	[200,250]	$NE(a_4) = [0.3,0.45] * [160,210] + [0.1,0.25] * [250,300] + [0.6,0.75] * [200,250] = [193,357]$
$NP(\theta_j)$	[0.3,0.45]	[0.1,0.25]	[0.6,0.75]	

By comparing the elements of the column $NE(a_i)$ we notice that:

$$NE(a_k) = \text{Max}_i [NE(a_i)] = [340,532.5] = NE(a_1)$$

It is corresponding to the alternative a_1 then the alternative a_1 is the appropriate decision according to this rule.

3. Conclusion and Results:

In any matter of making a decision, we rely on the data that we can obtain through reality, and if these data are classic values, the decision we will take will be in accordance with those data, and any change that occurs during the work cannot be controlled and may lead to unexpected losses, this cannot happen if the data are neutrosophic values, because these values have a margin of freedom that takes into account all the changes that may occur during the course of work from the worst conditions to the best, and the decision that we adopt based on these values is an appropriate decision for all circumstances, the following is a comparison between the results obtained when using the rules of decision-making in the case of risk according to the classical logic and neutrosophic logic:

Table 14: Compare the results

Largest expected values	Most probably rule	Aspiration level rule		
a_1	a_3	a_1	The appropriate alternative	Classical logic
340	500	400	The profit	
a_1	a_2	a_1	The appropriate alternative	Neutrosophic logic
[340,532.5]	[500,550]	[400,450]	The profit	

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