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An Investigation in the Initial Solution for Neutrosophic Transportation Problems (NTP)

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Abstract.

Transportation issues arise often in everyday life. To ensure that the regions' needs for transported material are met at the lowest feasible cost, materials must be carried from production centers to consuming centers as quickly as possible. Operations research approaches, notably mathematical programming, are utilized to solve these recurring and daily challenges. The problem's data is transformed into a mathematical model, and then the best solution is discovered using the proper procedures. When dealing with transportation issues, we arrive at a linear mathematical model, which can be solved using the direct simplex method and its modifications. However, because of the clarity and specificity of the transportation model, scholars and researchers were able to find other methods that were easier than the simplex method.

Whatever method is used, the goal is to determine the number of units transferred for any material from the production centers to the consumption centers in order to minimize transportation costs, keeping in mind that each export center has its own capacity and cannot supply quantities of the material greater than that capacity. Furthermore, each import center has a certain requirement for which it makes a request and for which it is unable to consume further quantities. In this manuscript, the researchers will use the North-West Corner approach, the Least-Cost method, and Vogel's approximation method to discover an initial solution to the balanced neutrosophic transport problems.

The term "neutrosophic transportation problems (i.e. NTP)" refers to the transportation problems in which the required and available quantities have neutrosophic values of the form $Na_i =$

$a_i + \varepsilon_i$, where ε_i is the indeterminacy in the produced quantities, and it is either $\varepsilon_i = [\lambda_{i1}, \lambda_{i2}]$ or $\varepsilon_i = \{\lambda_{i1}, \lambda_{i2}\}$. While the required quantities are also neutrosophic values of the form $Nb_j = b_j + \delta_j$, here δ_j is the indeterminacy on the required quantity, and it is either $\delta_j = [\mu_{j1}, \mu_{j2}]$ or $\delta_j = \{\mu_{j1}, \mu_{j2}\}$. It is worthy to mention that when the problem is unbalanced (i.e. the summation of the required quantities is not equal to the produced quantities), then firstly will convert the problem to a balanced one.

In this article, the authors assumed the representation of the neutrosophic numbers as intervals such as $\varepsilon_i = [\lambda_{i1}, \lambda_{i2}]$, $\delta_j = [\mu_{j1}, \mu_{j2}]$. It is important to notice that the authors did not adopt (trapezoidal numbers, pentagonal numbers, or any other neutrosophic numbers which need to specify using the membership functions, this kind of neutrosophic numbers or parameters represented by intervals have been firstly introduced by Smarandache F. in his main published books [32,33].

The sections of this manuscript has been organized as follow: the introduction is the inception of this article, section one has been dedicated to the north-west corner method containing three subsections for three case studies depending upon the existence of the indeterminacy in the problem, the least- cost method represents section two, while section three has been devoted to Vogel's approximation method, section four is conclusion and results.

Keywords: Transportation Problem; Neutrosophic Transportation Problem (NTP); Initial Solution; Consumption Centers (CC); Production Centers (PC); Available Quantities (AQ); Required Quantities (RQ); North-West Corner Method; Least-Cost Method; Vogel's Method.

Introduction

Transportation problems are among the most prevalent linear programming problems encountered in everyday life. These problems study the transfer of materials from production centers to consumption centers in the shortest period of time or at the lowest cost, or the distribution of transportation modes (such as buses, planes and ships etc.) on the imposed transportation lines in which the requests can meet by the least cost, and since the mathematical models we obtain are linear models, so the simplex method and its modifications can be used to obtain the optimal solution, but the special nature of the transportation problems enabled scientists to find special ways to solve these models that dependent on finding initial solution and then using other ways to improvement this initial solution using heuristics algorithms to find optimal solution [1-5], there are previous studies for transport models at the lowest cost using neutrosophic environments, which is the new vision of modelling and is designed to effectively address the uncertainties inherent in the real world, as it came to replace the binary logic that define merely the truthiness status and falseness status, by introducing a third, neutral state which can be interpreted as undetermined or uncertain .

This neutrosophic logic has been established in 1995 by the philosopher and mathematician Florentin Smarandache [7,9,10,11,13] introduced as a generalization to both: fuzzy logic presented by L. Zadeh in 1965 [6], and intuitionistic fuzzy logic introduced by K. Atanassov in 1983 [8]. In addition, A. A. Salama presented the theory of classical neutrosophic sets as a generalization of the classical sets. Theory [12,20], he developed, introduced and formulated new concepts in the fields of topology, statistics, computer science... etc. through neutrosophic theory [15,17-19, 22,28,29].

The neutrosophic theory has grown significantly in recent years through its application in measurement theory, group theory, graph theory and many scientific and practical fields [8,14-16, 21, 23-27,30-35], this research sheds the light on the modified that same methods used to find the initial solution for the classic transport problems in finding a preliminary solution to the neutrosophic transport problems in its three forms, the first form is that form when the cost is neutrosophic values, while the second form occurs when the demanded quantities and the supplied quantities are neutrosophic values, finally, the third form occurs when the cost of transport and the demanded quantities and the supplied quantities are all neutrosophic values.

Discussion:

It is popular that there are several ways to find a basic (initial) solution to the transfer problem, given the condition that the number of the basic variables in this initial solution must equal the number of the linear conditions (i.e. m+n-1). An initial solution can be found in several ways. In this research, we will use three methods, the North-West corner method, the least-cost method, and Vogel's approximate method, to find an initial solution to the neutrosophic transport model, and we will study the three forms in each of the methods. [1-5].

1. The North-West Corner Method

The North-West Corner Rule is a technique for calculating an initial basic feasible solution to a transportation problem. The method is called North-West Corner because the basic variables are chosen from the extreme left corner. the following three steps gives the initial basic feasible solution:

1. Find the north west corner cell of the transportation tableau. Allocate as much as possible to the selected cell, and adjust the associated amounts of supply and demand by subtracting the allocated amount.
2. Cross out the row or column with 0 supply or demand. If both a row and a column have a 0, cross out randomly row or column.
3. If one cell is left uncrossed, cross out the cell and stop. Otherwise, go to step 1.

We should not forget that the final allocated cells (nonzero cells) must equal to the value (m+n-1), where m determine how many production center that the problem have, while n refers to how many consumption center exist.

1.1 First Case Study for Neutrosophic Transportation Problem in which the Indeterminacy is in Nc_{ij}

In this case the cost of transportation will be neutrosophic values, that's mean the monetary value of transfer one unit from the production center i to the consumption center j is $Nc_{ij} = c_{ij} \pm \varepsilon$, where ε is the indeterminate value and equal to $\varepsilon = [\lambda_1, \lambda_2]$, so the payment matrix will be $Nc_{ij} = \lfloor c_{ij} \pm \varepsilon \rfloor$. Although all transportation costs have been given the same indeterminate value, it is feasible to assign a different indeterminate to each cost and use the same case study.
The Text of the Problem
A certain amount of oil is to be transported from three stations A_1, A_2, A_3 to four cities B_1, B_2, B_3, B_4.The following table shows the quantities available at each station, the demand quantities in each city,and the transportation costs in each direction:

PC \ CC	$B1$	$B2$	$B3$	$B4$	**AQ**
$A1$	$7+\varepsilon$ $x11$	$4+\varepsilon$ $x12$	$15+\varepsilon$ $x13$	$9+\varepsilon$ $x14$	120
$A2$	$11+\varepsilon$ $x21$	$0+\varepsilon$ $x22$	$7+\varepsilon$ $x23$	$3+\varepsilon$ $x24$	80
$A3$	$4+\varepsilon$ $x31$	$5+\varepsilon$ $x32$	$2+\varepsilon$ $x33$	$8+\varepsilon$ $x34$	100
RQ	85	65	90	60	

In this example, the indeterminate value of ε = [0,2], based on the problem's data we have, $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 300$, this signifies that the issue is balanced. Substituting ε = [0,2] into the preceding tableau yielded the following one:

PC \ CC	B_1	B_2	B_3	B_4	**AQ**
A_1	[7,9] x_{11}	[4,6] x_{12}	[15,17] x_{13}	[9,11] x_{14}	120
A_2	[11,13] x_{21}	[0,2] x_{22}	[7,9] x_{23}	[3,5] x_{24}	80
A_3	[4,6] x_{31}	[5,7] x_{32}	[2,4] x_{33}	[8,10] x_{34}	100
RQ	85	65	90	60	300 \ 300

The entire calculations to find the initial solution using the North-West method have been summarized as follow:

Start with the cell located in the north-west corner of the table, i.e. that cell corresponds to the first production center crossing with the first consumption center, this cell will carry the value

Min {85,93} = 85, hence the first consumption center B_1 has the need been fulfilling of its requirement from the first production center A_1, the remaining amount in $A_1$is 120 − 85 = 35. Move to the right cell positioned in the crossing of first row with second column and put in it the value Min {65,35} = 35, so the available quantity in A_1, and the required quantity in B_1 both are being zero, but the second consumption center B_2 requires to 65 − 35 = 30. Go down to the cell of position in the cross of second row with second column (x_{22}) and put the value Min {30,80} = 30, consequently, the second consumption center B_2 has its need been fulfilling, and the remaining value in the second production center A_2 is 80 − 30 = 50, keep going in the same above technique till all production centers are emptied, and all consumption centers have been fulfilling, finally the following table was gotten:

PC \ CC	B_1	B_2	B_3	B_4	AQ
A_1	[7,9] 85	[4,6] 35	[15,17]	[9,11]	120
A_2	[11,13]	[0,2] 30	[7,9] 50	[3,5]	80
A_3	[4,6]	[5,7]	[2,4] 40	[8,10] 60	100
RQ	85	65	90	60	300 \ 300

The last table illustrates: $x_{11} = 85, x_{12} = 35, x_{22} = 30, x_{23} = 50, x_{33} = 40, x_{34} = 60, x_{13} = x_{14} = x_{21} = x_{24} = x_{31} = x_{32} = 0,$ we have $n = 4, m = 3, m + n - 1 = 6,$ meaning that the initial conduction satisfied the necessary condition. Calculate the total cost for this initial solution by substitution the $x's$ values in the cost function:

$$NC = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} + c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33} + c_{34}x_{34}$$

$$NC = [7,9] * 85 + [4,6] * 35 + [15,17] * 0 + [9,11] * 0 + [11,13] * 0 + [0,2] * 30 + [7,9] * 50 + [3,5] * 0 + [4,6] * 0 + [5,7] * 0 + [2,4] * 40 + [8,10] * 60 = [1645,2245]$$

Which is the cost versus initial solution.

1.2 Second Case Study in which the indeterminacy is in both production center and consumption center

Problem Text

A quantity of fuel is intended to be shipped from three stations to four cities. The available quantities at each station, the demand quantities in each city, and the transportation costs in each direction are demonstrated in the following table:

PC \ CC	$B1$	$B2$	$B3$	$B4$	**AQ**
$A1$	7	4	15	9	$120+\varepsilon 1$
$A2$	11	0	7	3	$80+\varepsilon 2$
$A3$	4	5	2	8	$100+\varepsilon 3$
RQ	$85+\delta 1$	$65+\delta 2$	$90+\delta 3$	$60+\delta 4$	

It is worthy to mention that $\varepsilon_1, \varepsilon_2, \varepsilon_3$ represent the indeterminacies that exist in available quantitiesin the fuel stations, and it can be took as intervals $[\lambda_{i1}, \lambda_{i2}]$ or as sets $\{\lambda_{i1}, \lambda_{i2}\}$ …etc.

For this case study, the following values have been picked: $\varepsilon_1 = [0,11], \varepsilon_2 = [0,9], \varepsilon_3 = [0,15]$

While the values δ_1, δ_2, δ_3, δ_4 are the indeterminacies in the required quantities in the four cities,also these neutrosophic values can be regarded as intervals $[\mu_{i1}, \mu_{i2}]$ or as sets $\{\mu_{i1}, \mu_{i2}\}$ …etc.
For this example, the following neutrosophic values have been took: $\delta_1 = [0,8], \delta_2 = [0,12], \delta_3 = [0,9]$, $\delta_4 = [0,6]$. So the above table becomes:

PC \ CC	$B1$	$B2$	$B3$	$B4$	**A**
$A1$	7	4	15	9	[120,131]
$A2$	11	0	7	3	[80,89]
$A3$	4	5	2	8	[100,115]
RQ	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] / [300,335]

It is clear that the problem is balanced as $\sum_{i=1}^{3} Na_i = \sum_{j=1}^{4} Nb_j = [300,335]$.

Start with the cell located in the north-west corner of the table, i.e. that cell corresponds to the first production center crossing with the first consumption center, this cell will carry the value Min {[85,93], [120,131]} = [85,93], hence the first consumption center $B1$ has the need been fulfilling of its requirement from the first production center $A1$, the remaining amount in $A1$ is [120,131] − [85,93] = [35,38]. Move to the right cell positioned in the crossing of first row with second column and put in it the value Min {[65,77], [35,38]} = [35,38], so the available quantity in $A1$, and the required quantity in $B1$ both are being zero, but the second consumption center $B2$ requires to [65,77] − [35,38] = [30,39]. Go down to the cell of position in the cross of second row with second column ($x22$) and put the value Min {[30,39], [80,89]} = [30,39], consequently, the second consumption center $B2$ has its need been fulfilling, and the remaining value in the second production center $A2$ is [80,89] − [30,39] = [50,50], keep going in the same above technique till all production centers are emptied, and all consumption centers have been fulfilling, finally the following table was gotten:

This table contains the following neutrosophic values:

$Nx_{11} = [85,93], Nx_{12} = [35,38], Nx_{22} = [30,39], Nx_{23} = [50,50], Nx_{33} = [40,49], Nx_{34} = [60,66], Nx_{13} = Nx_{14} =$

PC \ CC	$B1$	$B2$	$B3$	$B4$	**AQ**
$A1$	7 [85,93]	4 [35,38]	15 0	9 0	[120,131]
$A2$	11 0	0 [30,39]	7 [50,50]	3 0	[80,89]
$A3$	4 0	5 0	2 [40,49]	8 [60,66]	[100,115]
RQ	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] \ [300,335]

$Nx_{21} = Nx_{24} = Nx_{31} = Nx_{32} = 0$. In this problem similar to the previous problem in section (1.1), $n = 4, m = 3 \Longrightarrow m + n - 1 = 6$, meaning that the initial condition satisfied the necessary condition. Calculate the total cost for this initial solution by substitution the x's values in the cost function:

$NC = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} + c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33} + c_{34}x_{34}$

$NC = 7 * [85,93] + 4 * [35,38] + 15 * 0 + 9 * 0 + 11 * 0 + 0 * [30,39] + 7 * [50,50] + 3 * 0 + 4 * 0 + 5 * 0 + 2 * [40,49] + 8 * [60,66] = [1645,1779]$

Which is the cost versus to the initial solution.

1.3 Third Case Study in Which the Transportation Cost, the Available Quantities in the Production Centers, and the Demand Quantities in the Consumption Centers are all Neutrosophic Values

By taking the same context of the studied cases in the previous subsections (1.1, 1.2) subject to the following table has been considered as new example:

PC \ CC	$B1$	$B2$	$B3$	$B4$	**A**
$A1$	$7 + \varepsilon$ $Nx11$	$4 + \varepsilon$ $Nx12$	$15 + \varepsilon$ $Nx13$	$9 + \varepsilon$ $Nx14$	$120 + \varepsilon1$
$A2$	$11 + \varepsilon$ $Nx21$	$0 + \varepsilon$ $Nx22$	$7 + \varepsilon$ $Nx23$	$3 + \varepsilon$ $Nx24$	$80 + \varepsilon2$
$A3$	$4 + \varepsilon$ $Nx31$	$5 + \varepsilon$ $Nx32$	$2 + \varepsilon$ $Nx33$	$8 + \varepsilon$ $Nx34$	$100 + \varepsilon3$
RQ	$85 + \delta1$	$65 + \delta2$	$90 + \delta3$	$60 + \delta4$	

By assuming $\varepsilon = [0,2], \varepsilon_1 = [0,11], \varepsilon_2 = [0,9], \varepsilon_3 = [0,15], \delta_1 = [0,8], \delta_2 = [0,12], \delta_3 = [0,9], \delta_4 =$

[0,6], the above table can be rewritten as:

CC / PC	B_1	B_2	B_3	B_4	AQ
A_1	[7,9]	[4,6]	[15,17]	[9,11]	[120,131]
A_2	[11,13]	[0,2]	[7,9]	[3,5]	[80,89]
A_3	[4,6]	[5,7]	[2,4]	[8,10]	[100,115]
RQ	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] / [300,335]

Obviously, the problem is balanced because $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = [300,335]$. The same north-west strategy has been implemented to get the following table:

CC / PC	B_1	B_2	B_3	B_4	AQ
A_1	[7,9] [85,93]	[4,6] [35,38]	[15,17] 0	[9,11] 0	[120,131]
A_2	[11,13] 0	[0,2] [30,39]	[7,9] [50,50]	[3,5] 0	[80,89]
A_3	[4,6] 0	[5,7] 0	[2,4] [40,49]	[8,10] [60,66]	[100,115]
RQ	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] / [300,335]

Same as the previous examples we have, $Nx_{11} = [85,93], Nx_{12} = [35,38], Nx_{22} = [30,39], Nx_{23} = [50,50], Nx_{33} = [40,49], Nx_{34} = [60,66], Nx_{13} = Nx_{14} = Nx_{21} = Nx_{24} = Nx_{31} = Nx_{32} = 0$. In this problem similar to the previous problems (1.1 &1.2), n = 4, m = 3 $\Longrightarrow$ $m + n - 1 = 6$, meaning that the initial condition satisfied the necessary condition. Calculate the total cost for this initial solution by substitution the x's values in the cost function:

$$NC = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} + c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33} + c_{34}x_{34}$$

NC = [7,9] $*$ [85,93] + [4,6] $*$ [35,38] + [9,11] $*$ 0 + [7,9] $*$ 0 + [11,13] $*$ 0 + [0,2] $*$ [30,39] + [7,9]* [50,50] + [4,6] $*$ 0 + [5,7] $*$ 0 + [2,4] $*$ [40,49] + [8,10] $*$ [60,66] = [1645,2449]

Which is the cost versus to the initial solution.

2. The Least- Cost Method

The least cost is another method used to obtain the initial feasible solution for the transportation problem. Here, the allocation begins with the cell which has the minimum cost. The lower cost cells are chosen over the higher-cost cell with the objective to have the least cost of transportation. The Least Cost Method is considered to produce more optimal results than the north-west corner because it considers the shipping cost while making the allocation, whereas the North-West corner method only considers the availability and supply requirement and allocation begin with the extreme left corner, irrespective of the shipping cost [1-5]. We will discuss the existence types of the indeterminacies either in transportation costs, or the indeterminacy exists in both the available quantity in the production centers and in the demand quantities in the consumption centers, or in all of them by the following case studies:

2.1 Case Study Has the Indeterminacy in its Transportation Cost

The same context of example (1.1) has been resolved using least- cost method, where the least cost cell is [0,2] which is located in the position resulting from the intersection of the second row with the second column, and put the value $min\{65,80\} = 65$ in it. Thus, we have met the need of the second consumption center B_2 from the second production center A_2, and the remaining quantity in A_2 is $80 - 65 = 15$. Move to the next least cost value among the remaining costs is [2,4], which is located in the cell resulting from the intersection of the third row with the third column, and we put the value $min\{90,100\} = 90$ in it. Thus, we have met the need of the third consumption center B_3 from the third productive center A_3, and the remaining value in the third productive center is $100 - 90 = 10$. Continuing by the same strategy until all consumption centers have been saturated and all production centers have been emptied. Consequently, the following table yield:

CC / PC	B_1	B_2	B_3	B_4	AQ
A_1	[7,9] 75	[4,6]	[15,17]	[9,11] 45	120
A_2	[11,13]	[0,2] 65	[7,9]	[3,5] 15	80
A_3	[4,6] 10	[5,7]	[2,4] 90	[8,10]	100
RQ	85	65	90	60	300 / 300

Here, $x_{11} = 75$, $x_{14} = 45, x_{22} = 65, x_{24} = 15$, $x_{31} = 10, x_{33} = 90$, $x_{12} = x_{13} = x_{21} = x_{23} = x_{32} = x_{34} = 0$.
NL = [7,9] $*$ 75 + [4,6] $*$ 0 + [15,17] $*$ 0 + [9,11] $*$ 45 + [11,13] $*$ 0 + [0,2] $*$ 65 + [7,9] $*$ 0 + [3,5] $*$ 15 + [4,6] $*$ 10 + [5,7] $*$ 0 + [2,4] $*$ 90 + [8,10] $*$ 0 = [1195,1795]. As usual, it represents the cost versus to the initial solution.

2.2 Case Study in Which the Available Quantities of the Production Centers and the Demanded Quantities of the Consumption Centers are Neutrosophic Values

For the comparison purposes, the same data and problem text that used in the case study (1.2) has been considered here, so the first three tables are the same. By applying the least cost method, it seemsthe cell of the zero value is the required cell which exactly located in the intersection of the second row with the second column, so we put in this cell the value min {[65,77], [80,89]} = [65,77], by moving to the next least cost cell that located in the intersection of the third row with the third columnof 2 value, burden this cell with the value min {[90,99], [100,115]} = [90,99], go on with the same strategy till all consumption centers saturation at the same time all productions centers have been emptied, hence, the following table yielding:

CC / PC	B_1	B_2	B_3	B_4	AQ
A_1	7 [75,77]	4	15	9 [45,54]	[120,131]
A_2	11	0 [65,77]	7	3 [15,12]	[80,89]
A_3	4 [10,16]	5	2 [90,99]	8	[100,115]
RQ	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] / [300,335]

Hence, Nx_{11} = [75,77], Nx_{14} = [45,54], Nx_{22} = [65,77], Nx_{24} = [15,12], Nx_{31} = [10,16], Nx_{33} = [90,99], Nx_{12} $= N_{13} = Nx_{21} = Nx_{23} = Nx_{32} = Nx_{34} = 0$, we should not forget the problem satisfiesthe balancing condition since $m + n - 1 = 6$.

$NL = 7 * [75,77] + 4 * 0 + 15 * 0 + 9 * [45,54] + 11 * 0 + 0 * [65,77] + 7 * 0 + 3 * [15,12] + 4* [10,16] + 5 * 0 + 2 * [90,99] + 8 * 0 = [1259,1323]$

Obviously, it represents the cost versus to the initial solution.

2.3 Case Study in Which the Transportation Cost, the Available Quantities of the Production Centers, and the Demanded Quantities of the Consumption Centers are all Neutrosophic Values

In this section, the text problem and the types of the indeterminacies are same as in the case study ofthe section (1.3), but the values of $\delta_i's$, $\varepsilon_i's$ are assumed to be:
ε = [0,2], ε_1 = [0,35], ε_2 = [0,10], ε_3 = [0,15], δ_1 = [0,7], δ_2 = [0,18], δ_3 = [0,25], δ_4 = [0,10].

CC / PC	B_1	B_2	B_3	B_4	AQ
A_1	[7,9] Nx_{11}	[4,6] Nx_{12}	[15,17] Nx_{13}	[9,11] Nx_{14}	[120,155]
A_2	[11,13] Nx_{21}	[0,2] Nx_{22}	[7,9] Nx_{23}	[3,5] Nx_{24}	[80,90]
A_3	[4,6] Nx_{31}	[5,7] Nx_{32}	[2,4] Nx_{33}	[8,10] Nx_{34}	[100,115]
RQ	[85,92]	[65,83]	[90,115]	[60,70]	[300,360] / [300,360]

Again, by using the least cost strategy, the least cost is the cell [0,2] located in the cell intersect the second row with second column, so we will put the value min{65,80} = 65 in it, hence the needy of the second consumption center $B2$ has been met from the second production center A_2 , the remaining quantity in A_2 is 80 − 65 = 15 , by moving to the next least cost cell which is [2,4] represents the cell allocated in the intersection of the third row with the third column, burden this cell with the value min{90,100} = 90 . Thus, we have met the need of the third consumption center B_3 from the third productive center A_3, and the remaining value in the third productive center is 100 − 90 = 10. Go on with the same strategy till all consumption centers saturation at the same time all productions centers have been emptied, hence, the following table yielding:

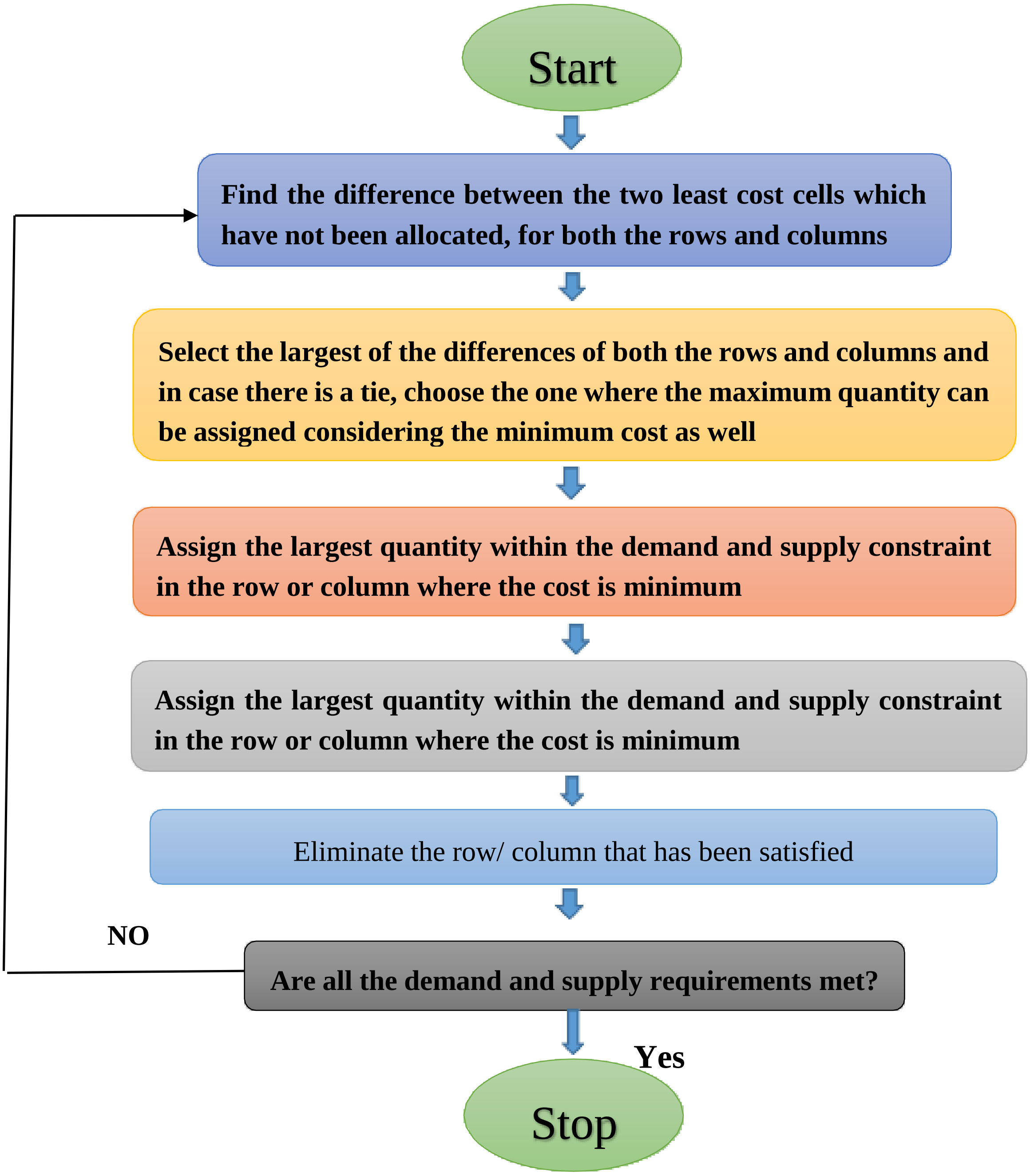
CC / PC	B_1	B_2	B_3	B_4	AQ
A_1	[7,9] [75,77]	[4,6]	[15,17]	[9,11] [45,54]	[120,131]
A_2	[11,13]	[0,2] [65,77]	[7,9]	[3,5] [15,12]	[80,89]
A_3	[4,6] [10,16]	[5,7]	[2,4] [90,99]	[8,10]	[100,115]
RQ	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] / [300,335]

Hence, Nx_{11} = [75,77], Nx_{14} = [45,54], Nx_{22} = [65,77], Nx_{24} = [15,12], Nx_{31} = [10,16], Nx_{33} = [90,99], $Nx_{12} = Nx_{13} = Nx_{21} = Nx_{23} = Nx_{32} = Nx_{34} = 0$, we should not forget the problem satisfies the balancing condition since $m + n - 1 = 6$. The following cost represents the initial feasible solution.

NL = [7,9] ∗ [75,77] + [4,6] ∗ 0 + [15,17] ∗ 0 + [9,11] ∗ [45,54] + [11,13] ∗ 0 + [0,2] ∗ [65,77] + [7,9] ∗0 + [3,5] ∗ [15,12] + [4,6] ∗ [10,16] + [5,7] ∗ 0 + [2,4] ∗ [90,99] + [8,10] ∗ 0 = [1195,1993].

3- Vogel's Approximation Method :

Definition: The Vogel's Approximation Method or VAM is an iterative procedure calculated to findout the initial feasible solution of the transportation problem. Like Least cost Method, here also the shipping cost is taken into consideration, but in a relative sense. The following is the flow chart showing the steps involved in solving the transportation problem using the Vogel's Approximation method.



The same cases studies that have discussed in the all previous sections (1.1,1.2,1.3,2.1,2.2,2.3) can be presented here with the same problems texts, with same data values, to be resolved in the Vogel's iterative method, this will give us a good opportunity to analyze the results which enables us to make a good comparison between (North-West Corner method, Least-Cost Method, and Vogel's Method).

3.1 Case Study Has the Indeterminacy in its Transportation Cost

CC / PC	B_1	B_2	B_3	B_4	AQ	Δ_1	Δ_2	Δ_3
A_1	[7,9] 75	[4,6] 45	[15,17]	[9,11]	120	3	3	3
A_2	[11,13]	[0,2] 20	[7,9]	[3,5] 60	80	3	3	11
A_3	[4,6] 10	[5,7]	[2,4] 90	[8,10]	100	2	2	1
RQ	85	65	90	60	300 / 300			
Δ'_1	3	4	5	5				
Δ'_2	3	4	5	_				
Δ'_3	3	4	_	_				

The symbols $\Delta'_1, \Delta'_2, \Delta'_3$ mean the subtractions between the columns respectively, while the symbols $\Delta_1, \Delta_2, \Delta_3$ mean the subtractions between the rows respectively.

From the above table, we have $x_{11} = 75, x_{12} = 45, x_{22} = 20, x_{24} = 60, x_{31} = 10, x_{33} = 90, x_{13} = x_{14} = x_{21} = x_{23} = x_{32} = x_{34} = 0,$

$$NL = [7,9] * 75 + [4,6] * 45 + [15,17] * 0 + [9,11] * 0 + [11,13] * 0 + [0,2] * 20 + [7,9] * 0 + [3,5] * 60$$
$$+ [4,6] * 10 + [5,7] * 0 + [2,4] * 90 + [8,10] * 0 = [1105,1705]$$

3.2 Case Study in Which the Available Quantities of the Production Centers and the Demanded Quantities of the Consumption Centers are Neutrosophic Values

CC / PC	B_1	B_2	B_3	B_4	AQ	Δ_1	Δ_2	Δ_3
A_1	7 [75,77]	4 [45,54]	15	9	[120,131]	3	3	3
A_2	11	0 [20,23]	7	3 [60,66]	[80,89]	3	7	–
A_3	4 [10,16]	5	2 [90,99]	8	[100,115]	2	2	2
RQ	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] [300,335]			
Δ'_1	3	4	5	5				
Δ'_2	3	4	5	–				
Δ'_3	3	1	13	–				

Recall the same notations $Nx_{ij}'s$ with their new values,

$$Nx_{11} = [75,77], Nx_{12} = [45,54], Nx_{22} = [20,23], Nx_{24} = [60,66], Nx_{31} = [10,16], \quad Nx_{33} = [90,99],$$

$$Nx_{13} = N_{14} = Nx_{21} = Nx_{23} = Nx_{32} = Nx_{34} = 0$$

The initial feasible solution is:

$$NL = 7 * [75,77] + 4 * [45,54] + 15 * 0 + 9 * 0 + 11 * 0 + 0 * [20,23] + 7 * 0 + 3 * [60,66] + 4 * [10,16] + 5 * 0 + 2 * [90,99] + 8 * 0 = [1105,1215]$$

3.3 Case Study in Which the Transportation cost, the Available Quantities of the Production Centers, and the Demanded Quantities of the Consumption Centers are all Neutrosophic Values

Recall the same text problem in section (1.3) with respect to resolving it using Vogel's iterative procedure to conclude the following table:

CC / PC	B_1	B_2	B_3	B_4	AQ	Δ_1	Δ_2	Δ_3
A_1	[7,9] [75,77]	[4,6] [45,54]	[15,17]	[9,11]	[120,131]	3	3	3
A_2	[11,13]	[0,2] [20,23]	[7,9]	[3,5] [60,66]	[80,89]	3	7	−
A_3	[4,6] [10,16]	[5,7]	[2,4] [90,99]	[8,10]	[100,115]	2	2	2
RQ	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] [300,335]			
Δ'_1	3	4	5	5				
Δ'_2	3	4	5	_				
Δ'_3	3	1	13	_				

So the initial feasible solution is:

$$NL = [7,9] * [75,77] + [4,6] * [45,54] + [15,17] * 0 + [9,11] * 0 + [11,13] * 0 + [0,2] * [20,23] + [7,9] * 0 + [3,5] * [60,66] + [4,6] * [10,16] + [5,7] * 0 + [2,4] * [90,99] + [8,10] * 0 = [1105,1885]$$

4. Conclusion and Results

This paper sheds the light on a new vision for solving Transportation problems by taking into consideration the existence of indeterminacy in many joints of the problems that have been solve nine times, each time in different method and different aspect of indeterminacy existence. With somedeep insights the reader can notice that the Vogel's iterative procedure yields minimum cost than both the costs that produced by applying North-west method, and Least-Cost method. However, themethods that used is still gives the results regarded as the initial feasible solution not the optimal solution, which mean that these methods are still need to improve to get the optimal solution.

Below table summarizes all previous solutions in comparison strategy:

The Method / Types of the problem	North-West Method	Least- Cost Method	Vogel's Method
First type indeterminacy	[1645,2245]	[1195,1795]	[1105,1705]
Second type indeterminacy	[1645,1779]	[1259,1323]	[1105,1215]
Third type indeterminacy	[1645,2449]	[1195,1993]	[1105,1885]

Looking forward to the further upcoming studies dedicated to implement improvements on theinitial solutions to get an optimal solution in the neutrosophic transportation problems.

As it is well known in the transportation problems, that the (North- West Corner, Least Cost Method, and Vogel's Method), all these methods are for finding the initial solutions (either these initial solutions are suffering from weak accurate by applying the North - West Corner, or having more accurate by applying Vogel's method), it still needs to investigate the optimal solutions in the transportations problems, which will be by intending to publish forthcoming papers.

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